Review of
Missouri K-12 Mathematics: Core Content, Learning Goals and Performance Indicators.
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Summary and Recommendations:

I received an invitation from DESE to review the revised version [2] of the *K-12 Mathematics Core Concepts, Learning Goals, and Performance Indicators.* I was one of many Missouri mathematicians who had requested in May 2008 [8] that DESE include mathematicians as an integral (and preferably not subsidiary) part of the process of fixing problems with the original version of the Missouri document [1]. Four mathematicians [13,15,17,18] had been previously invited by DESE to review the original document [1], as had a large number of educators. I have had access to these mathematicians' reviews, as well as to subsequent comments by some of these mathematicians [14,16], but I have not had access to the educators' reviews. I have also had access to the Report of the Writing Group [12], which summarized the comments received on the first version [1] and the resulting modifications in the revised version [2].

Summary of Mathematical Content Review:

Here is a summary of my review, and a synthesis of the relevant parts of the reviews by other mathematicians, of the mathematical content of the revised document [2]:

1) Grades K-7 are have been significantly improved in the revision. Many of the criticisms made for Grades K-7 of the previous version have been successfully addressed by the writing group in the revision. I present my own comprehensive review of the revised Grades K-7 below, and propose several relatively minor modifications which would in my opinion result in satisfactory standards for Grades K-7.

2) Grade 8 has been somewhat improved in the revision, but serious problems remain. The writing group has *not* successfully addressed many of the key criticisms made for grade 8 in the previous version. I present my own comprehensive review of Grade 8 below, as well as citations to material from previous reviewers which remains relevant. I propose specific, but fairly extensive, modifications which could in my opinion result in satisfactory standards for Grade 8.

3A) The Algebra 1 - Geometry - Algebra 2 sequence constitutes one of the two parallel tracks for grades 9 and higher. This material has undergone only very minor changes in this revision, and remains extremely problematic. In particular, I strongly disagree with the Writing Group's implied claim [12] that the document satisfactorily covers the "Major Topics of School Algebra" as per the unambiguous recommendation of the National Mathematics Advisory Panel (NMP) [3]. I will refer primarily to the previous reviewers as relevant for this material, since the material had not changed sufficiently to require a new comprehensive review. I do not even start to propose specific modifications to fix this material, rather I am forced to agree with previous reviewers [13] and [16] who urge that this material be scrapped and new material written from scratch.

3B) I have not reviewed the Integrated track in grades 9 and higher, and will not comment on this material.

Mathematical Content Standards and Pedagogical Methodology Standards:

It is necessary to acknowledge that both mathematical content and pedagogical ideology can play a major role in a document on the teaching and learning of mathematics. It is well-known that arguments about pedagogical ideologies have risen to the level of "math wars" at various locales and times over the last two decades. The growing realization among the leading experts that the math wars have been unproductive and need to stop has led in the last few years to the following very important documents constituting the convergence towards a consensus and truce in the math wars:

- The 2006 NCTM Focal Points for grades K-8 [4]

- The 2008 report [5] of the National Mathematics Advisory Panel (NMP or NMAP) for preparation and implementation of authentic high-school algebra

The NCTM Focal Points represents a major evolution from the previous NCTM documents, which had almost exclusively dictated *pedagogical standards* rather than *mathematical content standards*. The NCTM Focal Points constitutes a widely accepted set of mathematical "focal points" for grades K-8, while simultaneously advocating for the "investigations based" (or "student-centered") pedagogical focus that had been consistently advanced by the prior NCTM documents. The NMP report represents a further evolution of the consensus position, firmly advising against the prescription of a particular pedagogical method ("student centered" or "teacher centered") because the panel's comprehensive review of the literature in education research failed to provide any evidence supporting the preference of one or the other method. The mathematical standards of states such as California [6] and Massachusetts [7], which place at the top of the 2005 Fordham ranking [5] of state mathematics standards, also explicitly and repeatedly disavow the prescription of a particular teaching ideology.

Missouri, which placed at the very bottom of the 2005 Fordham rankings, continues to differ in a major way from the top ranked state standards and from the NMP recommendations in retaining a pervasive bias towards the "investigations-based" ("student-centered") pedagogical ideology in the state math standards. The Missouri document under review [2] does not in any way disavow or minimize this bias. In fact, the five primary "strands" of the Missouri document are *pedagogical*, while the primary "strands" of the California and Masschusetts documents are *mathematical content*. This is a problem because the mathematical content is consequently accorded lower priority in the Missouri document than the relentless promotion of "investigations based" pedagogical methods at every stage. At the lower grades this is sustainable, though not necessarily advisable, but at the higher grades this becomes unsustainable. Essential mathematical content in Algebra 1, Geoemtry, and Algebra 2 (and even Grade 8) is significantly simplified and/or deferred in order to have students "discover" their own concepts and procedures "in a variety of ways" rather than defining essential concepts or practicing standard procedures.

Determining the appropriate balance between *mathematical content standards* and *pedagogical methodology standards* is ultimately an administrative/political decision for DESE and the Missouri Board of Education. If the decision is made that pedagogical methodology should dominate, then a separate decision needs to be made about which type of pedagogy is to be promoted. Finally, a decision needs to be made about the relative importance of meaningfully implementing the recommendations of the 2008 National Mathematics Advisory Panel report.

I will end this section by venturing my own personal opinion. I believe that Missouri has already gone way too far in letting pedagogical methodology dominate mathematical content, and that the "investigations based" pedagogy that has been actively promoted in Missouri education documents is seriously misguided and outdated. My own local Columbia Public School District should serve as a warning to the rest of Missouri in this regard. The Columbia Public School District had several years ago adopted the very curricula being so manifestly promoted in the current document (Investigations-TERC for all students in K-5, CMP for most students in 6-8, Core-Plus for the integrated track in High School). Suffice it to say that the results have been a disaster in every conceivable way, and the Superintendent has been forced to make the drastic (but correct) decision of summarily banning several of these curricula.

Recommendations for Revising the Missouri Standards:

I'm afraid that I have to stand by the recommendations made in the mathematicians' letter of May 2008. This letter had requested that the decisions about the state math standards be made by a group that include members knowledgeable in mathematics, and who are sympathetic to the goal of advancing Missouri mathematics standards into the modern era. Although the current writing group has improved the grade K-7 standards by adhering closely to the Focal Points and the advice of outside expert reviewers, they have

proved themselves unable and/or unwilling to make any notable improvements on the higher grades. Simply asking the current writing group to again revise a document that they already find satisfactory is not a viable option. Further, it will become increasingly difficult to find reviewers willing to read and compare successive iterations which differ primarily in reshuffling and rearrangement of the same ideas.

It is essential that Missouri students have access to a solid mathematics education through high school, an education that would allow them to study technical areas at the college and university level and beyond. In order to achieve this goal, **I propose that a separate and independent writing group be formed to completely rewrite the standards for the Algebra 1 - Geometry - Algebra 2 sequence, and to work on grade 8 jointly with the current writing group.** The current writing group would remain in charge of the lower grades, and of the Integrated track in high school. This represents a win-win situation for all parties, in particular Missouri schools could offer state of the art instruction in both Algebra - Geometry, and separately in "investigation based" reform math. Students, teachers, and parents could make appropriate individual decisions based on a student's career goals, educational goals, and learning style.

Review of Mathematical Content:

General Overview of the Mathematical Content - Grades K-8:

The mathematical content standards have been improved in the lower grades in close accord with the NCTM Focal Points. With relatively minor modifications these standards are acceptable through grade 7. However, the improvement seriously stalls at grade 8, and would require extensive rewriting to ensure students are ready for an *authentic* sequence of Algebra 1 - Geometry - Algebra 2 as specified by the NMP. Unfortunately the Algebra 1- Geometry - Algebra 2 sequence in the Missouri standard is very far off the mark, and needs to be rewritten completely to bring it in line with the NMP recommendations.

It is significant that Algebra 1 in 8th grade is is the norm for students in leading states like California and Massachusetts. Algebra 1 (or Integrated 1) does not start until the 9th grade in the Missouri standards, although some students take an accelerated sequence. The NMP sets out a major goal of increasing the number of students taking an authentic Algebra course in 8th grade. Since readiness for authentic algebra is such an important goal, to ensure that all students adequately learn the required material by 8th grade it would be reasonable to significantly reduce some of the auxiliary standards like data analysis and probability in the earlier grades. It would further be reasonable to think of grade 8 as a year primarily focusing on "pre-algebra" or "fundamentals of algebra", and to provide additional assessment and study opportunities for students who need extra help catching up to the required level in Grade 8. Some students may need additional help in Grade 7 as well as Grade 8 to prepare for success in an authentic Algebra 1 - Geometry - Algebra 2 sequence. It should be a high priority to make this option available to *all* students who are interested, and to advise students and families about the NMP findings about the importance of this option for students who are considering continuing to a college or university education. Identifying and helping individual students who are capable of high achievement at this stage of their studiest should help to address achievement gap issues as well, and in a way which is better attuned to the needs individual students than the common strategy of condemning groups of students to permanently lowered expectations.

To ensure that students in the early grades are learning the "Critical Foundations" required for later success in Algebra, the NMP report provides a list of 11 "Benchmarks" [3] (Table 2, p. 20) distributed over grade 3 through 7. I mark each of these 11 Benchmarks with the notation "NMP" in the following review of grades K-8, and comment about whether the Benchmark has been satisfactorily met by the proposed standards. Interpreting these Benchmarks flexibly as per the NMP recommendation, it is my opinion that 3 of the 11 Benchmarks are satisfactorily met, 3 are marginally met, and 5 are not satisfactorily met. The Report of the Writing Group [12] provides for each Benchmark a list of sections where "these topics are included in the Missouri document".

Although the revision of the Missouri document already (and rightly) reduced the amount of material on

data/data analysis/probability in the lower grades, this material could be safely cut further if additional time is required to focus on more fundamental mathematics. This could be at the discretion of school districts or individual teachers, giving additional focus to fundamentals for students who require this, while including the data analysis and related material as enrichment for students who have mastered the fundamentals. To achieve this flexibility it would be preferable to cut most the data analysis and related material from the standards. This material is accorded the following weights currently in grades K-8: Grade 2 CC-D (10 percent), Grade 4 CC-D (10 percent), Grade 5 CC-D (10 percent), Grade 6 CC-E (15 percent), Grade 7 CC-D (10 percent), grade 8 CC-D (15 percent). This adds up to 70 percent of a full academic year, of which much could be safely reclaimed for students who need additional focus on fundamentals. For the data analysis that does remain, it is important to focus on important concepts such as the mean, and to avoid mere exercises in "eyeballing" data or using sophisticated graphing calculator functions to "compute" regression coefficients which have not been properly defined or explained.

The number line, along with the standard order relations $<$ and $>$, should be accorded the central status it deserves as the most important and *coherent* representation of the integers, rational numbers, and real numbers. A variety of rather arbitrary representations, such as "10 x 10 grids", seem to be accorded equal or sometimes greater importance in the Missouri document. This results in the order relation being treated in the undesirable "spiraling" approach, sometimes just applied to measured lengths, sometimes to exploring facts about multiplication etc. It is not until grade 7 that students are expected to "compare and order integers, locate integers on a number line", but this is *way* too late. Why not specify the number line in early standards like grade 1 CC-A 1 b) which states: "read, write, compare and order numbers through 100" ? I'm concerned that the well-known (and to my mind completely unjustified) aversion to "standard algorithms" like long division seems to be spreading to "standard concepts" such as order on the number line. This would not be a good development, and would further slow the learning of important mathematics.

I couuld help but notice that a number of standards in the first version of the Missouri started with the phrase "Create and solve (simple) contextual problems ...." . In the second revision, many of these standards have been split into a pair of two separate standards. The first standard instructs the student to "Create contextual problems...." using certain content, then a separate standard is required to tell the student to "Solve contextual problems..." using exactly the same content as the preceding standard. This type of repetition only obfuscates the key ideas, and the chronic overuse of the words "context" and "contextual" don't help here or elsewhere.

Mathematical Content Review for the Revised Standards for each Grade K-8:

Kindergarten:

This is generally fine. It is consistent with Focal points at this grade level. (There are no NMP Benchmarks for this grade).

Grade 1:

This is generally fine. It is consistent with Focal points at this grade level. (There are no NMP Benchmarks for this grade).

CC B: The treatment of subtraction could be made simpler and clearer, for example following the suggestions of Prof. Milgram's review of the first draft (p.18).

Grade 2:

This could use some minor improvements. It is for the most part consistent with the Focal Points at this grade level. CC-D is not in the Focal Points at this level and could be eliminated here. (There are no NMP Benchmarks for this grade.)

CC A: Should include some very basic multiplication facts here, see Milgram's review (p.21). In CC-A 1 a), the standard "number line" is conspicuously absent in the "variety of representations of three-digit whole numbers".

The standard order comparisons $>$, $<$, $=$ had played a bigger role in the first version. The standard order $>$ has been reduced rather peripheral status in the locally assessed items CC-B 2 d) and CC-C 1 e).

CC C: As pointed out by Prof. Milgram, it is important to recognize "inexactness" in measurement as distinct from "estimation". It would be good to also discuss the transitive property of $>$ in CC C 1 e) as a "concept" on the number line to clarify any potential confusion that may occur from measurement-related complications.

CC-D This comprises 10 percent of Grade 2. If needed, this could be eliminated at this grade level to make room for a better treatment of more fundamental concepts. This is not in the Focal Points for this grade, nor is it in the NMP Benchmarks.

Grade 3:

This reasonably covers the Focal Points at this grade level. The material on perimeter in CC-D is more than what is indicated in the Focal Points at this grade level, but this material does appear in the fourth grade. It would be reasonable to postpone much or all of the study of perimeter until the following year, and use the additional 10 percent of the year (half of the 20 percent assigned to CC-D) to focus on a better understanding of the fundamentals at the third grade Focal Point level.

NMP: There is an NMP Benchmark at this grade level: "By the end of Grade 3, students should be proficient with the addition and subtraction of whole numbers." This NMP Benchmark seems to be met adequately in Gr 3 CC-A, as noted in the Report of Writing Group.

CC B: It would be a very good idea to actually define what is meant by division, in the manner suggested by Milgram (p. 26)

CC D: Perimeter is not in the Focal Points at grade 3. It is in the Focal Points at grade 4. The Grade 3 Connections to the Focal Points for Measurement do mention perimeter as a measurable attribute, but it does not play an essential conceptual role here. Gr 3 CC-D 3 should probably be moved to fourth grade CC-C (replacing the material on transformations Gr 4 CC-C 2.)

Grade 4:

The Focal Points for this grade level are reasonably covered. But there is a significant amount of material that could and should be removed from this grade in order to focus on fundamentals.

CC C 1: The area computations should include areas of TRIANGLES, not just rectangles. This is very fundamental, and needed to understand areas of polygons as specified in the Geometry Connections to the Focal Points at this grade level. Currently the areas of triangles are not mentioned until Grade 6 CC-C 2, along with far more advanced material on circles.

CC-C 2: The material on transformations that is now in Gr 5 CC-C 2 should be eliminated in this grade. Transformations are not in the 4th grade Focal Points, and cannot be covered in any meaningful manner at this level. The material on perimeter that had been in the 4th grade CC-D 3 could reasonably be moved here, replacing the material on transformations.

CC-D: This could be eliminated entirely. It is currently 10 percent of Grade 4. As discussed by Milgram (p. 38), this is not a good Data Analysis standard for this grade level, and is not in accord with the Data Analysis Connections to the Focal Points at Grade 4. If this is not eliminated, it needs to be significantly changed to include and emphasize frequency tables and the mean (the mean is introduced in Gr 6 CC-E).

The mode and median are far less important than the mean.

NMP There is an NMP Benchmark at this grade level: "By the end of Grade 4, Students should be able to identify and represent fractions and decimals, and compare them on a number line or with other common representations of fractions and decimals."

The Report of the Writing Group refers to Gr 3 CC-C (fractions) and Gr 4 CC-B (decimals). This NMP Benchmark is *not* adequately met. The "number line" representation is fundamental and should have a more prominent focus among the rather arbitrary "variety of representations" and "models" listed, which currently include "meter sticks, words, standard and expanded forms", and a "10 x 10 grid". The fundamental method to "compare" fractions or decimals is with the order inequalities $<$ and $>$, which are manifest on the number line but not in many of the other models. The "comparison" in the MO document are specified to just the extremely simple cases, such as "comparing fractions with like numerators or denominators".

Grade 5:

This grade level remains seriously muddled, and needs to be carefully rewritten. In particular, the aversion to actually defining any mathematical idea (as opposed to "discovering" it) is a serious problem at this grade. The first two Focal Points at this grade could be adequately covered with a sensible rewrite of this content, but are not in this version. The third Focal Point at this grade is not adequately covered, primarily because students have not yet covered the area of a triangle (this in Gr 6 CC-C) and are therefore limited to rather trivial shapes in two and three dimensions. CC-D on probability should be eliminated, this material cannot be adequately addressed at this level. The grade should focus on clarifying understanding and proficiency of what used to be called arithmetic.

CC A 1: The best way to "understand" division is to present and understand a definition and its consequences. It is not clear whether this standard intends for students to "understand" division or "practice" division, or something else altogether. Milgram suggested rethinking this standard completely, this was not substantively addressed, see Miglram's comments (p. 39)

CC B: This standard also remains very muddled. Milgram's detailed comments were not substantively addressed. A definition of fraction addition is called for here, see Milgram (p 37.)

CC C: Areas of triangles used to be here, have now been moved to Grade 6 CC-C. It would make much more sense to move areas of triangles to Grade 4 CC-C as suggested earlier. As it stands, students can only compute areas of rectangles at this point. The Focal Point specifies areas of polygons, requiring areas of triangles to decompose polygons. Similarly, they can only compute volumes in the very special case of prisms with rectangular bases, not prisms in general as specified by the Focal Point (Maybe they are expected to approximate volumes with "manipulatives" such as liquids or marbles and not know how to calculate these, this would be an unsatisfactory situation.)

CC D: Basic Probability Concepts (10 percent). This could be eliminated altogether. As pointed out by Milgram (p. 45), this is "terribly premature". The material could reasonably be subsumed into Gr 7 CC-D. For one, students need to be very good with fractions before even considering probability problems. Even the Data Analysis Connections to Focal Points at this grade only has only double-bar and line graphs.

There are three NMP Benchmarks at this grade level:

NMP By the end of Grade 5, students should be proficient with multiplication and division of whole numbers.

This seems ok.

NMP By the end of Grade 5, students should be proficient with comparing fractions and decimals and common percent, and with the addition and subtraction of fractions and decimals.

This requires sensibly rewriting CC-A and CC-B as discussed above. Currently this benchmark is *not* met.

NMP By the end of Grade 5, students should be able to solve problems involving perimeter and area of triangles and all quadrilaterals having at least one pair of parallel sides (i.e., trapezoids).

Th writing group report refers to Gr 4 CC-C (areas of rectangles); Gr 6 CC-C (perimeter and area of triangles, quadrilaterals, and circles.) As discussed above, this benchmark is *not* met, because areas of triangles are not covered until grade 6.

Grade 6: Grade 6 would benefit from shifting of some material between adjacent grades, but overall is ok. In particular, it is significantly better than grade 6 in the first edition.

CC-C: Much of this material should be shifted to other grades. The material on areas of triangles (and quadrilaterals) appears here for the first time. This basic and relatively easy topic should be moved to an earlier grade, specifically I suggest Grade 4 CC-C.

Significantly more advanced material on circles appears also in CC-C. The relation between the area or perimeter (i.e. circumference) of a circle is related to the radius or diameter by a formula that involves the irrational (and transcendental) number π. This is the reason the Focal Points defers this material in Grade 7, where it receives a significant amount of discussion, including polygonal approximations to circles representing rational approximations to the irrational number π. Grade 7 is where the material involving circles and π should be moved in this document as well. This leads nicely into the introduction of irrational numbers in grade 8.

Finally material from Grade 7 CC-C on volumes and surface areas not involving circles could be moved down to Grade 6 CC-C.

CC-D: This material on polynomial expressions and equations currently takes 15 percent of this grade. It might be a good idea to expand this material significantly in this grade, using some of the time gained from moving material from Gr 6 CC-C to other grades as discussed above, and as need from cutting down or eliminating Gr 6 CC-E as discussed below. Polynomials play an important role in the NMP "major topics" for Algebra.

CC-E: This material on Sample Survey and Data Distributions takes 15 percent of grade 6. The mean is introduced here. I had suggested considering the introduction of the mean earlier, Gr 4 CC-D, otherwise possibly eliminating Gr 4 CC-D. In that case, the Gr 6 CC-E could be eliminated. If the mean had not been introduced earlier, Gr 6 CC-E is a reasonable place to introduce it. In either case much of the non-mathematical material could be eliminated here.

There are three NMP Benchmarks at this grade level:

NMP By the end of Grade 6, students should be proficient with multiplication and division of fractions and decimals.

This Benchmarks seems ok.

NMP By the end of Grade 6, students should be proficient with all operations involving positive and negative integers.

Report of writing group refers to Gr 7 CC-A. This Benchmarks is *not* met, because inexplicably negative integers are not covered until Gr 7 CC-A. This year delay is very significant. This is one of the examples of where these standard seem to lose steam in the middle grades, with the effect that the students are unprepared for a meaningful high school curriculum.

NMP By the end of Grade 6, students should be able to analyze the properties of two-dimensional shapes and

solve problems involving perimeter and area, and analyze the properties of three- dimensional shapes and solve problems involving surface area and volume.

The writing group refers to Gr 6 CC-C for perimeter and area, and to Gr 7 CC-C for surface area and volume. This Benchmark is *not* met satisfactorily, although rather trivial cases involving rectangles are studied in Gr 5 CC-C. I had suggested above moving down material not involving circles from Gr 7 CC-C to Gr 6 CC-C, this would meet the Benchmark in a satisfactory manner.

Grade 7:

Linear equations (one independent variable) and relations to topics like proportionality are the major topic in the Grade 7 Focal Points, and this important material is also what is needed to ensure students are on track for High School Algebra as specified by NMP. The MO standards have some serious as they stand now are significantly flawed in covering this essential material , but they could be relatively easily fixed as outlined below. By shifting some of the more basic material about integers and areas/volumes to lower grades where it belongs, this grade can bring a much sharper needed focus on the interrelated concepts of rational numbers, linearity (over rational numbers), and proportionality.

CC-A: "Integers and Linear Equations". This standard mixes apples and oranges. LG-1 on integers, including negative integers, and the order relations $<$ and $>$ on integers, should have been covered before this grade level. It would be fully reasonable for LG-1 to cover the analogous material but more generally for rational numbers (positive and negative), not merely integers. Restricting to integers at this stage is too low.

CC-A LG-2 covers operations and problems with integers. This is reasonable at this grade level, but should *not* be in the same standard as LG-3 which covers linear relationships. Linearity is an extremely important concept at every stage of study. However, linearity does *not* really work over the integers, the full range of important features of linearity enter only when working over a field (such as the rational numbers, or the other common examples are real numbers or complex numbers.) Linear equations with integer coefficients cannot generally be solved with integers. Further, any linear equation with rational coefficients is equivalent to a linear equation with integer coefficients. It would make far more sense to change LG-3 b) to the more general case of rational coefficients where the full range of properties associated with linearity comes into play. It makes little sense to separately study linear equation with integer coefficients at this stage, and to revisit basically the same topic later (Gr 8 CC-A) in the guise of linear equations with rational coefficients.

CC-B: - First Focal Points for Grade 7 prominently specifies equations for proportionality, in particular $y = kx$ for proportionality and $xy = k$ for inverse proportionality. Equations should be explicit here as well. Milgram's comments on defining proportionality and slope in the revised document (p. 50) should be adopted here. Moving CC-A 3 to CC-B would be an improvement over its current pairing with CC-A 2 since linearity and proportionality are closely related, whereas linearity does not make sense over the ring of integers.

CC-C: As mentioned earlier, the material about areas of circles should be moved here, it is currently in Grade 6 CC-C. The material about volume and surface areas not involving circles should be moved down from here to Grade 6 CC-C. Approximating the area of a circle by decomposing into larger and larger numbers of wedges is important, because this is an excellent example of an irrational number represented as a series of rational numbers. It is particularly important to do this without introducing the inexactness and confusion of physical measurements by rulers or protractors.

CC-D: Experimental and theoretical probability, 10 percent. This is a Probability Connection to the Focal Points, seems ok here but not earlier. The material on probbility which I had suggested eliminating from Gr 5 CC-D could be included here. However, all of this could be optional if extra time is needed to properly cover more essential proportionality and linearity topics.

NMP By the end of Grade 7, students should be proficient with all operations involving positive and negative fractions.

The Report of writing group refers to Gr 7 CC-A. This benchmark is *not* met. As discussed above, Gr 7 CC-A is about the far simpler subject of ordering integers, serving as a very late introduction to negative integers. Order relations on the rational numbers (both negative and positive) are not discussed until Grade 8 CC-A, where they are mixed up with the much more complicated subject of real numbers. Although one cannot really expect a proper treatment of real numbers at this level, one can and must expect a proper treatment of the ordering of rational numbers in Gr 7 CC-A. This is in dire need of fixing. (Prof. Milgram's second review of the second version, Gr 8 page 53, appears to mistakenly assume that rational number had indeed been treated in grade 7 as they should have been.)

NMP By the end of Grade 7, students should be able to solve problems involving percent, ratio, and rate and extend this work to proportionality.

Writing group report refers to Gr 6 CC-B and Gr 7 CC-B. This is ok, *assuming* the fixes outlined above.

NMP By the end of Grade 7, students should be familiar with the relationship between similar triangles and the concept of the slope of a line.

Writing group report refers to Gr 7 CC-B 3. This is ok, *assuming* the fixes outlined above.

Grade 8:

Unfortunately the revised Missouri standards start to fall apart at Grade 8, and would require extensive modification to fix. The fixes required at Grade 8 are far more serious both practically and conceptually than the fixes outlined above for grades K-7.

CC-A: Real numbers, linear equations, inequalities and functions (35 percent). This standard is very seriously flawed, perhaps fatally flawed. Continuing and extending the Grade 7 focus on linear equations, notably including *systems of linear equations*, is the primary thread of the Focal Points at Grade 8, and constitutes essential content connecting with the Major Topics of School Algebra according the NMP.

CC-A LG-1: Most of this standard as it stands covers a equatios/function/inequality over the rational numbers. The bulk of the material on one linear equation over the rationals *should already have been covered* in Grade 7 CC-A and CC-B *assuming* that the arbitrary restriction to integer coefficients there had been removed as outlined above. The remaining standards are confusingly arbitrary. It's not clear why the "inequalities with rational coefficeints" considered in CC-A 1d) "involve non-linear relationships", while the equations in 1a) only involve "linear relationships". In CC-A 1g) considers "domain and range of a function". Rational functions provide the standard (and arguably best) examples for such problems, but rational functions have not been introduced yet.

As strongly stressed by Milgram in his review of the first version [13], and notably in his emphatic comments on the revised version [14] quoted below, the Grade 8 Focal Points expect far more than this extremely limited Missouri standard. Students should at this stage study *systems* of two linear equations in two unknows:

" I HAD POINTED OUT THE LACK OF ANY DISCUSSION OF TWO LINEAR EQUATIONS IN TWO UNKNOWNS IN MY PREVIOUS INVITED REVIEW AS A MAJOR FAILING IN THE GRADE 8 DRAFT STANDARDS, AND THE PRESENT VERSION, WHILE A DRAMATIC IMPROVEMENT ON THE PREVIOUS ONE, DOES NOT ADDRESS THIS. NOTE ALSO, THAT TWO EQUATIONS IN TWO UNKNOWNS ARE A PROMINENT PART OF THE NCTM EIGHTH GRADE FOCAL POINTS, SO THIS IS NOT, AT ALL, AN UNREASONABLE EXPECTATION."

CC-A LG-2: Some things are just loose ends that should have been resolved in earlier grades. For example CC-A 2 d) is (or at least should should be) implicitly covered in Gr 7 CC-A. More significantly, the standard

order relation on the rational numbers should have also been thoroughly covered in 7 CC-A as noted above. What then remains of LG-2 is extending the material from grade 7 on one linear equation over the field *rational* numbers to the analogous (actually identical) results about one linear equation over the field of *real* numbers.

In particular, LG- 2 a) states: "Recognize that the set of real numbers is made up of rational and irrational numbers and that they can be represented by points on a number line." Milgram second review page 55 says: *"More accurately, the number line is a model for the real numbers. More accurately yet, the number line is the real numbers. One does not have an actual mathematical object the number line until one has constructed the real numbers."* The number line is indeed the best and only model one has for the real numbers at this level. The characterization of rational numbers in terms of repeating decimal expansions seems implausible for Missouri students because students almost certainly will have been discouraged from developing sufficient facility in using in the standard algorithm for division to understand repeating decimal expansions. It is reasonable to get an impressionistic picture of irrational numbers by studying a few constructions/definitions of "numbers" that are demonstrably not rational (such as $\sqrt{2}$ or π). Although these are interesting, they are not particularly fundamental in the flow of important topics. Milgram's comment on LG-2, Milgram second review page 55 says: *"The material here is, for the most part, not too basic and receives far too much emphasis in these standards at the expense of material that is much more important, such as solving systems of two linear equations in two unknowns. So this material represents a serious dumbing down of the intent of the NCTM focal points in grade 8."*

(Mathematical aside: The standard order $<$ on the rational numbers plays a crucial role in the construction of the real numbers as the metric completion of the rational numbers, or the characterization of the real numbers as an ordered field with the Least Upper Bound property. Therefore the number line representation of rational numbers, for which the order $<$ is manifest, remains fundamental even through advanced undergraduate or beginning graduate studies in mathematics. Various arbitrary representations of numbers such as "10 x 10 grids" should be discouraged rather than encouraged at early levels.)

CC B: Transformations (15 percent). This material should be entirely eliminated at this stage. This had been stressed by Milgram in his first review [13], and even more unambiguously in his comments on the revised version [14] (p. 53):

"It is also worth noting that transformations are not nearly as important at this stage since it is virtually impossible to represent them rigorously without a great deal more expertise with linear equations and length and/or matrices as are topics like two linear equations in two unknowns. For example, understanding how two equations in two unknowns work and the types of solutions they have help prepare for a rigorous treatment of length and matrices as well as any number of of other topics critical to more advanced material in high school and college. So in your rush to get to a very specialized topic transformations you ignore the foundational material underlying it, and make both a very poor pedagogical and mathematical decision. I HAD POINTED OUT THE LACK OF ANY DISCUSSION OF TWO LINEAR EQUATIONS IN TWO UNKNOWNS IN MY PREVIOUS INVITED REVIEW AS A MAJOR FAILING IN THE GRADE 8 DRAFT STANDARDS, AND THE PRESENT VERSION, WHILE A DRAMATIC IMPROVEMENT ON THE PREVIOUS ONE, DOES NOT ADDRESS THIS. NOTE ALSO, THAT TWO EQUATIONS IN TWO UNKNOWNS ARE A PROMINENT PART OF THE NCTM EIGHTH GRADE FOCAL POINTS, SO THIS IS NOT, AT ALL, AN UNREASONABLE EXPECTATION. Also, it should be noted and emphasized that this material is NOT PRESENT in the grade 8 NCTM Focal Points, so it represents a serious dumbing down of the NCTM expectations, both by including material that cannot be adequately treated at this point, and so must be taught in a very sketchy, imprecise way, and by replacing much more basic and important material."

CC-C This is basically ok, consistent with second Focal Point in Grade 8.

CC D - Bivariate Data (15 percent). This should probably eliminated to make time for the careful study of more fundamental topics that many students will require at this crucial stage. If kept, one should be very

careful to avoid mindless "button pushing" on a calculator, using a calculator to compute some statistical measure that a student does not know how to define and is unable to compute without a calculator in even very simple situations.

There are no NMP Benchmarks for Grade 8, since preparation for Algebra would ideally be complete at grade 7, and Algebra would begin at grade 8. This is in fact what happens in the standards of leading states such as CA or MA.

General Overview of the Mathematical Content - Algebra I, Geometry, Algebra 2:

In contrast to the major revisions and significant improvements for grade K-8, the standards for Algebra I, Geometry, and Algebra 2 were not substantively revised or improved in the second version. With a handful of very minor exceptions such as those noted below, essentially of the comments made by the mathematicians reviewing the first version still apply to the revised version. In particular, Prof. Milgram's comprehensive review of the first version still stands as an outstanding (and damning) review of the revised version for these three courses. I have therefore not undertaken a comprehensive review of Algebra I, Geometry, and Algebra 2 in the revised document.

A sampling of a few of the the very minor changes I had noted between the first version and the second:

- Algebra 1 CC-A 2e): Parallel/perpendicular lines are now mentioned in Algebra 1, following the suggestion of s suggested Profs. Mitrea and Christiansen [15]

- Algebra 1 CC-C 2e) is new: "Represent simple exponential and quadratic equations using manipultive models, verbal description, tables and technology." Do students only "understand" the exponential function by manipulatives or technology? This is completely inadequate at this level.

- Algebra 1 no longer contains the CC: "Design statistical studies, recognize the importance of random sampling, identify sources of bias, and quantify the strength of linear relationships." Algebra 2 no longer contains the CC: "Distinguish among experiments, surveys, and observational studies". Other Data Analysis and Probability Core Contents, Algebra 1 Core Content D, Geometry Core Content D, appear to be rewritten to a much greater extent than the more fundamental CC's.

The reduction of these non-mathematical data standards is a minor positive development, but not offset by the abject lack of attention to the far more serious problems in the fundamental subjects of Algebra and Geometry. This does address one of the persistent, but not central, criticisms that had been directed at the first version of the document. Prof. Cheng had summarized this point well in his review [18]: *Data analysis and statistics are important, useful and applicable, and can be shown to students as inspiring applications of algebra. However, they do not rise to the level of requiring so much coverage in all of Algebra I, Geometry, and Algebra II. It is hard to understand why data analysis and statistics are included in Geometry, and it is also hard to accept so much coverage in both Algebra I and Algebra II if the intent is to show them as applications. Needles to say, time spent covering data analysis and statistics is time taken away from learning algebra. At a time when many students are not mastering algebra adequately, it is a major concern that precious time is diverted away.*

Profs. Mitrea and Christiansen confirm the overall absence of substantive improvement in their brief comments [16] comparing the original and revised versions of Algebra 1: *The Algebra I component of the revised document "Missouri K-12 mathematics: Core Content, Learning Goals and Performance Indicators" is a minor improvement over the Algebra I component in the previous version of this document. While the most egregious aspects we singled out in our previous review (which were just samples of how badly written the document was) were addressed, the document continues to be awed at multiple levels: it promotes low standards in our classrooms, omits critical topics, and provides little depth. Placing the new and old versions side-by-side shows only small changes - a few simple rearrangements and word additions. As such, they do not make this an acceptable document. Adoption of the proposed standards would be a disservice to the state*

of Missouri and will hurt its students for years to come.

Applicable Excerpts from the Original Reviews by Mathematicians:

I cite just a few of the many severe and comprehensive criticisms level by Prof. Milgram and Profs. Mitrea and Christiansen against the Algebra 1, Geometry, and Algebra 2 standards of the original document. The criticisms cited below, and many others, apply verbatim to the corresponding essentially unchanged sections of the revised document. I am personally in agreement with essentially all the points made by the mathematicians in their reviews of the original document. Note the high level of importance that these mathematicians ascribe to the Algebra recommendations made by the NMP.

Prof. Milgram [14] starts his comprehensive comments on the Algebra I CC-A as follows:

This is low level material and has already been asked for in grades 7 and 8. It was already pointed out in grade 8 that the Focal Points expect students to be able to solve TWO linear equations in 2 unknowns in grade 8. Consequently, Algebra I needs to be at a higher level. Let me make this more explicit. Here are the expectations for Algebra (Algebra I and II together) in the report of the National Math Panel:

(cites the "Major Topics" Table from NMP)

This lays out the Core Concepts of algebra about as well as one can do this. Note how very little it looks like what is currently here in these draft standards. Yes, it is true that these topics should be measured out and shared between Algebra I and Algebra II. But it is THESE TOPICS, not the vague philosophical discussions of the meanings of a few of these topics that we see in too many of the the draft standards here. Algebra, by its nature, is the set of calculational tools in mathematics. So calculation with these concepts is crucial, not the discovery of them. It is certainly helpful if students can understand the proofs of results like the quadratic formula or the expression of the coefficients in the binomial formula as quotients of factorials. It is also helpful if students are able to discover for themselves a few of the easier initial steps leading to these results. But it is not essential. It is essential that they understand and apply these techniques fluently.

Following several additional pages of detailed discussion, Prof. Milgram concludes his comments on Algebra I CC-A:

These considerations and those that will be discussed further on should be central in the revision of these Algebra I standards. One other note is worth pointing out. I complained about the weakening of the geometry standards in grades 7 and 8 earlier. In particular, the supression of the discussion of things like congruence and similarity which seem to have been replaced by some discussion of the elements of the Euclidean group was troubling. Here is what the National Math Panel report has to say:

'The proof that the definition of a slope of a line is independent of the choice of the two points depends on considerations of similar triangles. It is therefore vitally important that students be given the opportunity to become familiar with the basic facts of similar triangles before studying algebra. This should include the fact that corresponding sides of similar triangles are in proportion, or that if two triangles have two pairs of equal angles, they are similar. Students can defer the proofs of these theorems to a later course on Euclidean geometry, but they need to be comfortable using them. Students will commonly be asked to use certain theorems before they learn why the theorems are true, (e.g., lessons on the Pythagorean Theorem or the sum of angles of a triangle as 180 degrees). Mathematics learning does not have to be formally linear. '

'With the correct definition of slope available, students are in a position to understand the relationship between slopes of lines, and the concepts of parallelism and perpendicularity. This understanding has a strong bearing on the study of simultaneous linear equations in two variables. Using the precise definition of the graph of a linear equation of two variables, one can prove that the solution of a pair of simultaneous linear equations is the point of intersection of the graphs of the two equations. If the graphs are parallel, they do not intersect, and therefore there is no solution to the simultaneous equations. When the parallelism of the graphs is translated into the language of slope, students arrive at the criterion for the solvability of

simultaneous equations in terms of the determinant of the system of equations. '

Thus we see the intricate linkage of the core concepts indicated in the NCTM Focal Points with the material in a solid Algebra I course. What I would strongly recommend is that these standards, and all the following standards for Geometry, Algebra II and integrated mathematics be entirely revised in collaboration with professional mathematicians in the departments of mathematics at the top universities in Missouri.

Professors Mitrea and Christiansen [15] independently arrived at the the same overall conclusion in their review of Algebra I and Integrated I:

We strongly feel that they (The Missouri standards) should be rewritten from scratch, taking into account the recently released Final Report of the 2008 National Mathematics Advisory Panel.

A compendium of much additional material is available from my website missourimath.org.

Some Comparisons of the Report of the Writing Group with the NMP:

The Report of the Writing Group [12] attempted to tabulate and address comments and suggestions that had been submitted about the first version of the document. Apparently some of the more vehement comments I had cited above did not make the cut. The following excerpt concerns the Writing Group's handling of the very important NMP recommendations:

Comment/Suggestion: *The content emphasis should be aligned with national documents, including the NMAP recommendations.*

Response of the Writing Group: *Several Core Content areas were moved to more closely align with the NMAP recommendations. In most cases, the revised document aligns with the NMAP recommendations. In a few cases, where the grade placement of a topic in the NMAP recommendations was quite different than the current (Missouri 2004 GLEs) placement, the WG reviewed multiple national documents for additional guidance (NAEP framework, NCTM Curriculum Focal Points, College Board and Achieve Standards). For example, three-dimensional measurement (volume and surface area) is recommended for grade 6 in the NMAP report and Grade 8 in the current Missouri (2004) document. However, it is included as a grade 7 focus in the NCTM Curriculum Focal Points and in v2.0 of the Missouri GLEs. Therefore, the WG included it as part of Core Content in Grade 7.*

Note that the response of the Writing Group to the NMP is limited to strategically "moving" and "placing" topics within the document. Compare this with the prior admonition of Professors Mitrea and Christiansen [15]: *The K-12 METS standards should not be used as the working draft for the new document. Simple rearrangements or additions will not suffice to make this an acceptable document, at least for the upper grades.*

Further note that the Writing Group based its rearrangement decisions upon the perusal of "multiple national documents for additional guidance". For the specific topic cited, three-dimensional measurement (volume and surface area), the mathematical nuance of distinguishing between the more difficult cases that involve π and the easier cases that do not seem to have been overlooked in the "multiple national documents". See my comments on Grade 6 CC-C and Grade 7 CC-C for more detail on this particular topic. "Multiple national documents" were also invoked as the first justification for decisions regarding "appropriate placement of particular mathematical content".

In general, the Writing group's coverage of NMP Major Topics is something of a non sequitur. I will illustrate with the material on the quadratic equation, perhaps the signature topic of a first authentic algebra course.

From Mitrea-Christiansen review [15] of Algebra I : *The California and Massachusetts standards include the topics of completing the square and the quadratic formula along with factoring as techniques for solving a quadratic equation. The K-12 METS standards omit this but includes some discussion of exponential*

functions. It is not clear how this can be done in any meaningful way given that there is apparently no discussion of rational exponents.

The Report of the Writing Group reproduces the very important "Major Topics of School Algebra" from NMP Table 1 p. 20, and gives references to where these "topics are included in the Missouri document". I will focus on the following extremely fundamental group of four topics:

Quadratic Equations

- Factors and factoring of quadratic polynomials with integer coefficients

- Completing the square in quadratic expressions

- Quadratic formula and factoring of general quadratic polynomials

- Using the quadratic formula to solve equations

The first item is said to appear in Algebra 1 CC-C, and also in Algebra 2 CC-A (I will not include the corresponding Integrated sequence courses). The remaining three items appear only in Algebra 2 CC-A. It takes a moment to process the astounding fact that **the quadratic formula is not introduced until Grade 11 in the Missouri standards !**

Even at that incredibly late stage, the quadratic formula is covered extremely shoddily, as almost a grudging concession rather than one of the central tools of elementary algebra. From Algebra 2 CC-A:

LG 2 d): *Solve quadratic equations and inequalities by factoring, completing the square, and with technology. Interpret and justify solutions in terms of the original problem context and represent solutions graphically.*

LG 2 e): *Apply the quadratic formula to solve quadratic equations and interpret the nature of its roots.*

I am left speechless..... over and out.

Appendix: Miscellanea.

From NMP [3]:

23) All-encompassing recommendations that instruction should be entirely student centered or teacher directed are not supported by research. If such recommendations exist, they should be rescinded. If they are being considered, they should be avoided. High-quality research does not support the exclusive use of either approach.

From a 2007 report [11] issued by the Columbia Public Schools, Columbia MO:

The METS Alliance made the following recommendation, in order to improve the performance of all P-20 students, in its report to Governor Blunt in August 2006:
Improve METS curricula and assessments. Revise Missouris K-12 GLEs and assessments for mathematics and science to support focused, **inquiry-based instruction** *modeled on* **internationally recognized best practices**. *Ensure that* **collegiate-level** *METS curricula follow the same focused,* **inquiry-based instruction**.

From the California Mathematics Standards [6]:

The **five strands** in the Mathematics Content Standards (Number Sense; Algebra and Functions; Measurement and Geometry; Statistics, Data Analysis, and Probability; and Mathematical Reasoning) organize information about the key standards for kindergarten through grade seven. It should be noted that the strand of mathematical reasoning is different from the other four strands. This strand, which is inherently embedded in each of the other strands, is fundamental in developing the basic skills and conceptual understanding for a solid math? ematical foundation. It is important when looking at the standards to see the reasoning in all of them.

From the start of the revised Missouri Mathematics Standards [2]:

Mathematics proficiency includes **five interwoven and interdependent strands**: (Conceptual understanding, Procedural fluency, Strategic competence, Adaptive reasoning, Productive disposition.)

In the Missouri standards document [1]: Learning Goals and Performance Indicators for Grade 5, Core Concept B, Item 1a) states:

"Make connections among representations of fraction addition and subtraction situations with objects, diagrams, words, expressions and equations."

Prof. Milgram's comments [13] include: *"I do not know what any of this means. At best, it means something like, in representations of fractions such as on the number line or an area model explain what addition represents. (In the area model, adding areas, in the line model on the non-negative part, it means adding lengths). But the actual words make no sense. "*

The exact language persists in the revised version [2] of the Missouri standards document.

From Mitrea-Christiansen review [15]: *A goal of the METS alliance is to ensure 'that Missouri would continue to compete in the global market calling for inicreased skills in the areas of mathematics, engineering, technology, and science'. We find it surprising, therefore, that the authors of these standard did not learn any lessons from those states with thriving high-tech industries, such as California or Massachusetts.*

From Mitrea-Christiansen review [15]: *Another general observation is that the standards for Algebra I are written with heavy emphasis on modeling and interpretation while the mastery of algebra techniques is left as*

a secondary goal. Take, for example, the Summary of Core Concepts. In the the column Number/Algebra, the words 'Interpret, Model, Represent' are the key terms used and emphsized with 'Solve" left for last. Our experience with incoming freshmen at MU underscores the fact that more and more students come to college lacking fluency in algebraic computations.

From Prof. Cheng's review [18]: *There appears to be a de-emphasis on solving equations symbolically which is one of the major deficiencies students have in math courses. Most frequently stated in this document is "... solve ... in contextual situation" or "... solve ... in context" but not "... solve... equations/inequalities". This was pointed out by Dossey in his report. I understand that this is a curriculum document, and that concepts, learning, and performance need to be integrated together. I think that the writing group has the intention to balance them. However, procedural fluency is one of the five strands of mathematical proficiency cited in this document. In reading through the performance indicators, there appears to be a lack of emphasizing procedural fluency as part of the learning outcome. This is a concern of mine and many others.*

Also from [18]: *It is stated that the writing group has taken into consideration the availability of technology. It is unclear to me, though, how the performance indicators are to be measured with or without technology. It is noted that "with or without technology" is mentioned at various places. It is a noble intent not to prescribe how teachers should organize or enact instruction. However, this document bears the responsibility of making it clear how a student's mastery will be measured. Omission of such clarity will make this an incomplete document and possibly create unintended confusions and misunderstandings. I would also encourage DESE to be mindful of this responsibility.*

From NMP [3]: Recommendation: All school districts should ensure that all prepared students have access to an authentic algebra course and should prepare more students than at present to enroll in such a course by Grade 8. The word authentic is used here as a descriptor of a course that addresses algebra consistently with the Major Topics of School Algebra (Table 1, page 16). Students must be prepared with the mathematical prerequisites for this course according to the Critical Foundations of Algebra (page 17) and the Benchmarks for the Critical Foundations (Table 2, page 20).

McCarthy [17]: *Students starting college who have not taken calculus are at a disadvantage. They will be unable to take calculus-based courses as Freshmen, and will be behind many of their peers if they wish to pursue engineering or science. The proposed standards, eve if completed by 11th grade, still need a further year of precalculus before students are ready for calculus, so it would be impossible for a student in a school that taught these standards to take calculus in High School. Missouri should have the goal that the top quartile (at least) of students take a year of calculus in high school. Currently, the suggestions for Grades 6-8 seem to be somewhat light, and extra material could be added there.*

Also from McCarthy [17]: *It is not clear if the standards are supposed to be 'minimum competency expected of everyone' or 'average level - achieving grade level expectations'. I would like to have three sets of standards: Minimal, Average, and Advanced. ... The decision of whether to stream students in mathematics by ability may be one made by local school boards; but teachers must be aware that when teaching to a class of mixed ability (or perhaps more accurately of mixed background competencies) they must provide fresh stimulation to students once they have mastered a topic, even if this is self-directed by students through workbooks.*

Achievement Gap? From NMP [3]:

27) **Explicit instruction** with students who have mathematical difficulties has shown consistently positive effects on performance with word problems and computation. Results are consistent for students with learning disabilities, as well as other students who perform in the lowest third of a typical class. By the term explicit instruction, the Panel means that **teachers provide clear models for solving a problem type using an array of examples, that students receive extensive practice in use of newly learned strategies and skills**, that students are provided with opportunities to think aloud (i.e., talk through the decisions they make and the steps they take), and that students are provided with extensive feedback.

References:

[1] March version of document:

Missouri K-12 Mathematics: Core Concepts, Learning Goals and Performance Indicators - DRAFT March 28, 2008

$http://dese.mo.gov/divimprove/curriculum/documents/learning\_goals\_draft\_032808.pdf$

available on [9]

[2] August version of document:

Missouri K-12 Mathematics: Core Content, Learning Goals and Performance Indicators DRAFT August, 2008

$http://dese.mo.gov/divimprove/curriculum/newwebpages/MOK12MathRevisedLearningGoalsDRAFT8\_21\_08.doc$

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[3] NMP: Foundations for Success: Report of the National Mathematics Advisory Panel, 2008

$http://dww.ed.gov/topic/topic\_landing.cfm?PA\_ID=8\&T\_ID=20$

$http://www.ed.gov/about/bdscomm/list/mathpanel/reports.html$

[4] NCTM Focal Points 2006

$http://www.nctm.org/standards/content.aspx?id=270$

[5] Fordham Foundation report 2005

$http://www.edexcellence.net/detail/news.cfm?news\_id=338$

[6] California documents:

$http://www.cde.ca.gov./be/st/ss/documents/mathstandard.pdf$

$http://www.cde.ca.gov./be/st/$

[7] Massachusetts documents

$http://www.doe.mass.edu/omste/ca.html$

[8] Letter from Missouri mathematicians to DESE and MDHE, May 13, 2008:

Newspaper article: *Math professors seek change in state's K-12 math curriculum: Article by Elise Catchings: Published in the Columbia Missourian, June 10, 2008* and the letter and endorsements available at [9]

[9] My website: $http://missourimath.org/$

[10] Missouri METS report, 2006

$http://www.missourimets.com/docs/METSAllianceReport.pdf$

[11] Columbia Public Schools Secondary Mathematics Task Force report, 2007

http : *//www.columbia.k*12*.mo.us/reports/secmath.pdf*

[12] Report of the Writing Group, August 15, 2008. Impact of Feedback on March 31, 2008 DRAFT of *K-12 Mathematics Core Concepts, Learning Goals, and Performance Indicators*

[13] Milgram invited review of March version [1], June 2008, available on [9]

[14] Milgram comments on August version [2], available on [9]

[15] Mitrea-Christiansen invited review of March version [1], June 13, 2008, available on [9]

[16] Mitrea-Christiansen comments on August version [2], available on [9]

[17] McCarthy review of March version, June 4, 2008

[18] Cheng invited review of March version, June 15, 2008