Revised DRAFT Missouri K-12 Mathematics:

Core Content, Learning Goals and Performance Indicators

Fall, 2008

This revised draft version of the Missouri K – 12 Mathematics: Core Content, Learning Goals and Performance Indicators was presented to the Missouri State Board of Education on September 11, 2008. No action has been taken on the document therefore we are still awaiting directions on how to proceed. In the meantime, we wanted to make the most current draft version available. Additional updates and revisions will be made to include alignment to the Show-Me Standards and relevant assessment documents.

Missouri Department of Elementary and Secondary Education

Mathematics, Engineering, Technology and Science (METS) Alliance

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PREFACE

Since the Outstanding Schools Act of 1993, several documents have been developed to aid Missouri school districts in organizing curriculum to enable all students to achieve their potential. Most prominently, the *Show-Me Standards* (1996) identify broad content knowledge and process skills for all students to be successful as they continue their education, enter the workforce, and assume civic responsibilities.

In response to the federal No Child Left Behind Act of 2001, the Missouri Department of Elementary and Secondary Education created the first set of K-12 grade-level mathematics learning expectations (GLEs), published in 2004. This document, updated to version 2.0 in 2007, conveyed to Missouri educators and to test developers mathematics assessment specifications for specific grades and courses, K-12. That is, it conveyed the mathematics that would be the focus of state assessment and the mathematics that would be assessed locally. However, it was not developed to specify all the mathematics content that should be taught at each grade level or the extent to which specific mathematics content should be emphasized.

Rationale for the Development of K-12 Mathematics Learning Goals

In 2007 three factors led to the development of this new document. First, Governor Matt Blunt hosted a summit of key business stakeholders focused on ensuring that Missouri would continue to compete in the global market calling for increased skills in the areas of mathematics, engineering, technology, and science. A recommendation of Summit participants called for a revision of Missouri's K-12 GLEs and assessments for mathematics and science to support instruction of important mathematics content and process skills (problem solving, reasoning, communication, connections, and representations).

Second, the Missouri State Board of Education authorized a change in the required high school annual assessment – moving from a general grade 10 mathematics assessment to a series of end-of-course assessments in mathematics, beginning with Algebra I in 2008-09. This change necessitated a revision of the 2004 Missouri GLE document from a grade-based framework to a course-based framework at the high school level. At about the same time, state legislation led to work by the Department of Higher Education (DHE) to articulate college entrance expectations in mathematics and other subjects.

Third, national attention focused on strategies to strengthen K-12 mathematics programs. The commonly held belief that the U.S. mathematics curriculum lacks focus, coherence and depth documented by the Trends in International Mathematics and Science Study (TIMSS) provided the impetus for additional work in defining curriculum standards/learning goals. In response, several national organizations (Achieve, Inc., the College Board, the National Council of Teachers of Mathematics and the American Statistical Association) developed model K-12 curriculum standards for mathematics and/or statistics. In 2006 the National Mathematics Advisory Panel was established to advise President George W. Bush and Secretary of Education Margaret Spellings on the best use of scientifically based research on the teaching and learning of mathematics. The final report of the Panel, released in March 2008, included a summary of mathematics content needed to learn algebra and be prepared for studying higher levels of mathematics. Thus, a wealth of new information and recommendations regarding the K-12 school mathematics curriculum were available in 2007 to guide local and state curriculum work.

These factors led to collaboration between the Missouri Department of Elementary and Secondary Education (DESE) and the Mathematics, Engineering, Technology and Science (METS) Alliance. As a joint initiative, DESE and METS collaborated to develop kindergarten through high school mathematics content standards that specify priorities (core content), learning goals and performance indicators for each grade (K-8) and core high school courses (Algebra 1, Geometry, Algebra 2, Precalculus and Integrated Mathematics 1, 2, 3, and 4). This document is the result of that collaboration which included input from a variety of constituents (teachers, school administrators, curriculum developers, mathematicians, and mathematics educators).

Overview of the Project

Beginning in November 2007 a writing group comprised of Missouri mathematics teachers, curriculum specialists and higher education representatives reviewed several national publications by groups including Achieve, Inc., the College Board, the American Statistical Association, and the National Council of Teachers of Mathematics. Each national group made recommendations about mathematics curriculum emphasis and organization based on extensive analysis of the needs of today's workforce and preparation for college. At the same time, the National Mathematics Advisory Panel (NMAP) commissioned by President George W. Bush was developing recommendations for K-12 school mathematics. Together with the Missouri GLEs (v. 2.0) and the National Assessment of Educational Progress (NAEP) framework, the recommendations of all of the national groups were used to craft the curriculum recommendations in this document.

A primary goal of the work was to articulate a focused, coherent, and forward-looking mathematics program to prepare all Missouri students for careers and livelihood in today's technology- and statistics-rich environment. Care was taken to outline a few content priorities (core content) per grade level or course rather than attempt to cover a wide variety of topics across all mathematics strands. However, mathematics from each strand (number/algebra; geometry/measurement; and data analysis/statistics) were given attention across K-12. In some cases (e.g., algebra in middle school) a strand is given priority while in other cases (e.g., data analysis/statistics in grade 1) a strand is not a content priority but can be used to support the development of other strands (e.g., number/algebra).

In addition to specification of content emphasis, this document emphasizes the development of important mathematical processes such as problem solving, reasoning, communication, connections and representations. Without these processes, the skills and understandings outlined in the content strands are of limited value.

Writing Group

This document was developed by a team of Missouri educators (K-12 teachers, curriculum supervisors, and representatives of higher education) with assistance from support staff at DESE and METS. The individuals who played key roles in this project include:

Writing Group Co-Chair: Cindy Bryant, Missouri DESE
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Review Process

A Draft of the document was posted on the Missouri DESE website in late March 2008 and all interested individuals and groups were invited to provide feedback. In addition, state and national experts were invited to provide a review of the Draft. Over 300 pages of suggestions and comments were received. The Writing Group met in the summer of 2008 to review the feedback and to modify the Draft in response to feedback and to the recommendations included in the final report of the National Mathematics Advisory Panel report released in March 2008.

The organizing agencies (DESE and METS) and members of the Writing Group offer their sincere appreciation to everyone who offered feedback on the DRAFT. In particular, we note the following individuals who provided feedback at the request of DESE and METS:

Yungchen Cheng, mathematician, Missouri State University
Linda Coutts, district mathematics coordinator, Columbia Public Schools
Juli Dixon, member, Florida K-12 Mathematics Standards Writing Group, University of Central Florida
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Vena Long, former DESE Mathematics Consultant, University of Tennessee
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Gary Martin, mathematics educator, Auburn University
John McCarthy, mathematician, Washington University
Jim Milgram, mathematician, Stanford University
Dorina Mitrea (with Tanya Christionsen), mathematician, University of Missouri
Janie Schelack, chair of the NCTM Curriculum Focal Points Writing Group, Texas A&M
Sue Sundberg, mathematics educator, University of Central Missouri
Marianne Weber, district mathematics coordinator, Webster Groves School District

The writing group commissioned by DESE and METS believes that every Missouri student should have access to a high-quality mathematics education. The mathematics content priorities, learning goals and performance indicators outlined here are offered to advance that goal.

Cindy Bryant
Barbara Reys
Co-Chairs, K-12 Mathematics Learning Goals Writing Group

INTRODUCTION

This document presents curriculum priorities (core content), learning goals and performance indicators for K-12 mathematics in Missouri schools. It outlines the mathematics all Missouri students are expected to learn, emphasizing the importance of *mathematics proficiency* (National Research Council, 2001) as the primary goal. Mathematics proficiency includes five interwoven and interdependent strands:

> *Conceptual understanding*—comprehension of mathematical concepts, operations, and relations.
>
> *Procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
>
> *Strategic competence*—ability to formulate, represent, and solve mathematical problems
>
> *Adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification
>
> *Productive disposition*—habitual inclination to see mathematics as sensible, useful, worthwhile, coupled with a belief in diligence and one's own efficacy. (NRC, 2001, p. 116).

The document outlines important mathematics ideas, concepts, skills and procedures that form the foundation for understanding and learning the subject of mathematics. It provides a framework to bring focus to teaching, learning, and assessing mathematics.

Emphasis on Mathematical Content

The document is formatted differently from Grade-level Learning Expectations (GLE) documents previously published by DESE. Rather than a matrix format (organized by grade and strand), this document is organized by core content (content priorities) at each grade and course, followed by learning goals, and then by performance indicators.

> *Core Content* – Important mathematical ideas/topics that serve as organizing structures for curriculum design, instruction, and assessment for a grade level or course.
>
> *Learning Goals* - A set of statements, organized around each core content area, indicating what students are expected to learn.
>
> *Performance Indicators* - Statements of specific and measurable learning outcomes.

The core content areas identified for each grade (K-8) and high school course represent a critical and unique element of this revision. They describe important mathematics to be learned and, along with the learning goals and performance indicators, build a cohesive curriculum K-12.

Performance Indicators have been reviewed to identify candidates for assessment at the state or local level. A performance indicator (PI) coded with an asterisk (*) is recommended for local, rather than state, assessment. A PI with no asterisk is a candidate for annual assessment at the state level (Grade 3 – 8 MAP Assessment or End-of-Course Exams).

Emphasis on Mathematical Processes

At every grade, students must be challenged to use mathematics to reason and solve problems and they must be provided opportunities to develop these mathematical processes. In addition, they must be able to communicate about mathematics including representing (in writing, speaking and various symbolic formats) mathematical ideas.

The National Council of Teachers of Mathematics (2000) describes five mathematical processes that are essential elements of K-12 mathematics. They include:

Problem Solving: Students learn to identify mathematical problems; to formulate and apply strategies to solve problems; and to convey solutions using appropriate terminology and mathematical representations.

Reasoning and Proof: Students learn to justify their thinking; to reinforce and extend logical reasoning abilities; to reflect on and clarify their thinking; and to ask questions to extend their thinking.

Communication: Students learn to solve problems by obtaining information through reading, listening and observing; to translate this information into mathematical language and symbols; to process this information mathematically; and to present results in written, oral, and visual formats.

Connections: Students learn to relate various mathematical ideas and to use these relations to expand the ways they can approach problems.

Representations: Students learn to create and use representations to organize, record, and communicate mathematical ideas; to select, apply, and translate among mathematical representations to solve problems; to use representations to model and interpret physical, social, and mathematical phenomena.

In this document mathematical processes are interwoven throughout the core content and learning goals so that they are not thought of in isolation but rather as part of learning the content of mathematics. Key ideas related to mathematical processes are also listed in the "Core Content Matrix" (see pp. 8-10) by grade band.

Statistical Literacy

As in the previous GLEs, this document includes a strand devoted to the development of statistical literacy. However, it is not a focus of attention at every grade level. Rather, it is included as a core content area in some grades. The inclusion of this strand is based upon recommendations from various national groups of the increased importance of statistics as a basic knowledge for citizenry and work environments. The American Statistical Association (2005) indicates:

> Statistical literacy is essential in our personal lives as consumers, citizens, and professionals ... Sound statistical reasoning skills take a long time to develop. They cannot be honed to the level needed in the modern world through one high-school course. The surest way to help students attain the necessary skill level is to begin the statistics education process in the elementary grades and keep strengthening and expanding students' statistical thinking skills throughout the middle- and high-school years. (p. 3).

Lynn Steen, former President of the Mathematical Association of America, describes the need for statistical (quantitative) literacy in *Mathematics and Democracy: The Case for Quantitative Literacy* (2001):

> Unfortunately, despite years of study and life experience in an environment immersed in data, many educated adults remain functionally innumerate...even individuals who have studied trigonometry and calculus often remain largely ignorant of common abuses of data and all too often find themselves unable to comprehend (much less to articulate) the nuances of quantitative inferences... Quantitative literacy empowers people by giving them tools to think for themselves, to ask intelligent questions of experts, and to confront authority confidently. These are the skills required to thrive in the modern world (pp. 1-2).

Statistical literacy is addressed in this document through a strand of core content that begins in the elementary grades and extends through the high school courses. While some high school students may

also take a specific course on statistics, the strand outlined in this document provides a basis for all Missouri students to acquire basic statistical literacy.

Technology and Mathematics Teaching and Learning

Technology, used in appropriate ways, can increase opportunities for student learning, enrich learning environments, and prepare students for an increasingly technology-rich work and life environment. While some of the mathematics outlined in this document will be taught and learned without the use of technology, other mathematical topics can be learned and used with the technology.

The content priorities, learning goals and performance indicators describe the mathematics students need to learn, not necessarily all of the tools that teachers might use to help students reach these goals. Teachers need to be knowledgeable about how technology can support students in learning mathematics, not replace their learning. More importantly, teachers need to know what mathematics students need to understand so that students can use the technology as well as make sense of the outputs from that technology.

Relationship to National Recommendations and Other DESE Documents

The grade and course level core content, learning goals and performance indicators are built upon the foundation of the *Show-Me Content and Process Standards* (1996). Other important resources in the development of this draft include the recommendations of the National Mathematics Advisory Panel (2008), the National Assessment of Educational Progress framework (2007), National Council of Teachers of Mathematics *Principles and Standards for School Mathematics* (2000) and *Curriculum Focal Points* (2006), Achieve, Inc. *Elementary Mathematics Benchmarks, Grades K-6* (2008) and *Secondary Mathematics Benchmarks, Grades 7-12* (2008), the American Statistical Association *Guidelines for Assessment and Instruction in Statistics Education* (2006*),* and the *College Board Standards for College Success* (2007). In addition, the 2008 (v2.0) Missouri *Mathematics Grade- and Course-Level Expectations* (GLEs) and the Department of Higher Education *Mathematics College Level Entrance Competencies* (Draft, 2008) were key resources.

Using the Document

The document outlines 3-5 key "core content" areas for each grade/course. Each core content area is further elaborated with statements of "learning goals" and "performance indicators." The document also includes an estimate of the intended relative emphasis of each core content area. That is, a percentage is noted for each core content area, ranging from 10% to 30%. The indicated percentages are estimates and should be considered in that light as local decisions are made. The sum of the percentages for each grade/course total 85-90% so that individual schools or school districts can modify the emphasis and/or add additional content in response to local needs.

As noted earlier, the core content and learning goals specified in this document are intended for ALL students, although some students may advance through the outlined content more quickly than others. In fact, it is common in many Missouri schools for some students in middle school to begin the formal study of algebra as outlined in the high school section of this report. As always, local educational agencies are encouraged to develop and implement policies for serving all students. It is acknowledged that modifications of the outlined content emphasis will be needed for students advancing at an accelerated pace.

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Summary of Mathematics Core Content:

Kindergarten through High School

Summary of Mathematics Core Content: Grades K-8

	Number/Algebra	Geometry/Measurement	Data/Probability	Problem Solving/Reasoning Communication/Connections/Representation
Gr. K	Counting (30%) Beginning Addition and Subtraction (25%)	Geometric Shapes and Spatial Relationships (20%) Informal Measurement (10%)		Use a variety of strategies to solve problems. Organize, record and describe in words mathematical thinking.
Gr. 1	Whole Number Relationships (25%) Addition and Subtraction: Operations and Beginning Basic Facts (40%)	Geometric Relationships (20%)		
Gr. 2	Base-ten Numeration and Place-Value (20%) Addition and Subtraction: Basic Facts Fluency and Multi-digit Computation (35%)	Linear Measurement (20%)	Categorical Data (10%)	
Gr. 3	Extending Addition and Subtraction (15%) Multiplication and Division: Operations and Beginning Basic Facts (25%) Meanings of Fractions and Fractional Relationships (25%)	Properties and Perimeter of Two-dimensional Figures (20%)		Apply and adapt a variety of strategies to solve problems. Make and investigate mathematical conjectures. Communicate mathematical thinking coherently and clearly to peers and teacher. Organize, record, communicate, and represent mathematical ideas.
Gr. 4	Multiplication and Division: Basic Facts Fluency and Multi-digit Computation (30%) Decimal Place Value and Connections to Decimals and Percents (30%)	Area Measurement and Transformations (15%)	Data Organization and Analysis (10%)	
Gr. 5	Division of Whole Numbers (30%) Addition and Subtraction of Fractions and Decimals (30%)	Properties of Three-dimensional Shapes, Volume and Surface Area (15%)	Basic Probability Concepts (10%)	
Gr. 6	Multiplication and Division of Fractions and Decimals (20%) Ratios, Rates and Percents (15%) Mathematical Expressions and Equations (15%)	Extending Properties and Measures of Two-dimensional Figures (20%)	Sample Surveys and Data Distribution (15%)	Apply and adapt a variety of strategies to solve problems. Select and use various types of reasoning and methods of proof. Communicate mathematical thinking coherently and clearly to peers, teachers, and others. Use the language of mathematics to express mathematical ideas precisely. Create, select, apply, and translate among mathematical representations to solve problems.
Gr. 7	Integers and Linear Equations (30%) Proportionality and Similarity (25%)	Surface Area and Volume (20%)	Experimental and Theoretical Probability (10%)	
Gr. 8	Real Numbers, Linear Equations, Inequalities and Functions (35%)	Transformations (15%) Distance and Angle Measurement (20%)	Bivariate Data (15%)	

Note: A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Summary of Mathematics Core Content: High School

Course	Number/Algebra	Geometry/Measurement	Data/Probability	Problem Solving/Reasoning Communication/Connections/Representation
Algebra 1	Linear and Nonlinear Relationships (15%) Linear Expressions, Equations, Inequalities and Functions (30%) Non-Linear Functions (30%)		Trend Lines and Correlation (10%)	Create, apply and adapt a variety of strategies to solve problems. Make and investigate mathematical conjectures. Develop and evaluate mathematical arguments and proofs. Use the language of mathematics to express mathematical ideas precisely. Understand how mathematical ideas interconnect. Create and use representations to organize, record, and communicate mathematical ideas. Select, apply, and translate among mathematical representations to solve problems and model situations.
Geometry		Geometric Reasoning, Proof and Representation (25%) Similarity and Congruency (25%) Direct and Indirect Measurement (25%)	Conditional Probability and Independence (10%)	
Algebra 2	Polynomial Expressions, Equations and Functions (30%) Exponential, Logarithmic and Other Functions (30%) Systems of Equations and Inequalities and Matrices (15%)		Binomial Expansion and Binomial Probability (10%)	

Integrated 1	Linear and Nonlinear Relationships (15%) Linear Expressions, Equations, Inequalities and Functions (25%) Nonlinear Functions (20%)	Geometric Reasoning and Relationships (15%)	Trend Lines and Correlation (10%)	Create, apply and adapt a variety of strategies to solve problems. Make and investigate mathematical conjectures. Develop and evaluate mathematical arguments and proofs. Use the language of mathematics to express mathematical ideas precisely. Understand how mathematical ideas interconnect. Create and use representations to organize, record, and communicate mathematical ideas. Select, apply, and translate among mathematical representations to solve problems and model situations.
Integrated 2	Systems of Linear Equations and Matrices (25%)	Geometric Proof, Similarity and Transformations (30%) Direct and Indirect Measurement (20%)	Conditional Probability and Independence (10%)	
Integrated 3	Polynomial Expressions, Equations and Functions (30%) Exponential, Logarithmic and Other Functions (25%) Sequences and Recursion (10%)	Indirect Measurement (10%)	Binomial Expansion and Binomial Probability (10%)	

Note: A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Core Content, Learning Goals and Performance Indicators

Grades K-5

KINDERGARTEN
Summary of Core Content

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Core Content A: Counting **30%**

Students develop an understanding of whole numbers, number patterns and relationships, and the concepts underlying counting. They recognize the number of objects in small groups with and without counting. They understand that number words refer to quantity, use one-to-one correspondence, and produce sets of given amounts. They use patterns with objects, sounds, and movements as a foundation for numeric relationships.

Core Content B: Beginning Addition and Subtraction **25%**

Students develop an understanding of the relationships among whole numbers and of joining and separating sets. They use numbers, including written numerals, to solve quantitative problems. They compare and order sets or numerals and understand that numbers occurring later in a number sequence are larger than those at the beginning of the number sequence. They model simple joining and separating situations with objects. They choose, combine, and apply effective strategies for answering quantitative questions, including comparing and ordering sets or numerals and counting the number in individual or combined sets.

Core Content C: Geometric Shapes and Spatial Relationships **20%**

Students develop an awareness of geometric shapes and spatial relationships. They interpret the physical world with geometric ideas (e.g., shape, orientation, spatial relations) and describe it with corresponding vocabulary. They identify, name, and describe a variety of two- and three-dimensional shapes presented in a variety of ways (e.g., different sizes or orientations). They use basic shapes and spatial reasoning to model objects in their environment.

Core Content D: Informal Measurement **10%**

Students develop an understanding of measurable attributes and use informal processes of measurement to compare and order objects. They compare the lengths of two objects both directly (by comparing them with each other) and indirectly (by comparing both with a third object), and they order several objects according to length.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

KINDERGARTEN

Kindergarten, Core Content A: Counting **30%**

Students develop an understanding of whole numbers, number patterns and relationships, and the concepts underlying counting. They recognize the number of objects in small groups with and without counting. They understand that number words refer to quantity, use one-to-one correspondence, and produce sets of given amounts. They use patterns with objects, sounds, and movements as a foundation for numeric relationships.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and apply the counting process to various situations.
   a) *Count forward at least to 31 and backward from 10.
   b) *Use one-to-one correspondence when counting a set of objects and keep track of counted and uncounted objects.
   c) *Know that the last counting word represents the number in the collection.
   d) *Produce sets of given amounts (e.g., show 6 objects).
   e) *Recognize that the rearrangement of objects in a set does not change the number of objects.

2. Understand and use patterns as a foundation for describing relationships among whole numbers.
   a) *Describe, extend and create sequential patterns (objects, sounds, movements, and numbers) based on repeating terms and recognize that some patterns may have many possible repeating terms.
   b) *Apply the 0-9 sequence when counting or writing numerals from one decade to the next decade.
   c) *Recognize that a teen number has one group of ten plus some ones.

Kindergarten, Core Content B: Beginning Addition and Subtraction **25%**

Students develop an understanding of the relationships among whole numbers and of joining and separating sets. They use numbers, including written numerals, to solve quantitative problems. They compare and order sets or numerals and understand that numbers occurring later in a number sequence are larger than those at the beginning of the number sequence. They model simple joining and separating situations with objects. They choose, combine, and apply effective strategies for answering quantitative questions, including comparing and ordering sets or numerals and counting the number in individual or combined sets.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand the relative magnitude of numbers at least to 31.
   a) *Represent numbers in a variety of ways and make connections between and among these representations by using objects, diagrams, words, and numerals.
   b) *Order numerals and recognize that numbers occurring later in a number sequence are larger than those at the beginning of the number sequence.
   c) *Compare sets of objects and determine whether they have the same, fewer, or more objects by matching and by counting.
   d) *Solve problems that involve comparing and ordering quantities.

2. Understand the meaning of addition and subtraction for whole numbers less than ten.
   a) *Represent and solve problems involving the joining and separating of sets by using objects, diagrams, words, and numerals.
   b) *Compose and decompose quantities in order to establish relationships between the parts and the whole and identify all pairs of whole numbers that make a given number (e.g., 0 and 5; 1 and 4; 2 and 3 all make 5).

Kindergarten, Core Content C: Geometric Shapes and Spatial Relationships 20%

Students develop an awareness of geometric shapes and spatial relationships. They interpret the physical world with geometric ideas (e.g., shape, orientation, spatial relations) and describe it with corresponding vocabulary. They identify, name, and describe a variety of two- and three-dimensional shapes presented in a variety of ways (e.g., different sizes or orientations). They use basic shapes and spatial reasoning to model objects in their environment.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand that two and three-dimensional shapes can be classified on the basis of their attributes.
   a) *Identify, name and describe two-dimensional shapes, including circles, triangles, rectangles, and squares.
   b) *Identify, name and describe three-dimensional shapes, including pyramids, rectangular prisms, cubes, cones, cylinders, and spheres.
   c) *Identify and describe shapes represented in the environment.
   d) *Sort objects into groups by attribute (shape or size) and identify which attribute was used.

2. Understand relative positions of objects in space.
   a) *Recognize that when shapes are moved (e.g., turned or shifted), they maintain the same classification.
   b) *Demonstrate relative positions in space (*over, under, above, below, on, beside, next to, and between*).

Kindergarten, Core Content D: Informal Measurement **10%**

Students develop an understanding of measurable attributes and use informal processes of measurement to compare and order objects. They compare the lengths of two objects both directly (by comparing them with each other) and indirectly (by comparing both with a third object), and they order several objects according to length.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand measurable attributes of objects through informal measurement.
   a) *Order objects directly or indirectly by using the measurable attributes of length.
   b) *Compare and describe the length of a group of objects (e.g., longer, longest, taller, tallest, shorter, shortest).

GRADE 1

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Grade 1, Core Content A: Whole Number Relationships 25%

Students develop an understanding of whole number relationships, including grouping in tens and ones. They compare and order whole numbers to develop an understanding of and solve problems involving the relative magnitude of these numbers. They think of whole numbers between 10 and 100 in groups of tens and ones (especially recognizing the numbers 11 to 19 as 1 group of ten and particular numbers of ones). They understand the sequential order of the counting numbers and represent numbers in a variety of ways.

Grade 1, Core Content B: Addition and Subtraction: Operations and Beginning Basic Facts 40%

Students develop an understanding of the operations of addition and subtraction, apply a variety of strategies for basic addition/subtraction facts, and relate addition and subtraction as inverse operations. They use a variety of models to develop an understanding of the meanings of addition and subtraction and strategies to solve arithmetic problems. They understand the connections between counting up and counting back with the operations of addition and subtraction (e.g., adding two is the same as "counting on" two). They use properties of addition (identity, commutative and associative) to add whole numbers, and they create and use increasingly sophisticated strategies based on these properties to solve addition and subtraction problems involving basic facts. By comparing a variety of solution strategies, children relate addition and subtraction as inverse operations.

Grade 1, Core Content C: Geometric Relationships 20%

Students develop an understanding of the relationships among geometric shapes. They compose and decompose plane and solid figures (e.g., by putting two congruent isosceles triangles together to make a rhombus), thus building an understanding of part-whole relationships as well as the properties of the original and composite shapes. By combining figures, they recognize them from different perspectives and orientations, describe their geometric attributes and properties, and determine how they are alike and different. Through these processes, they develop a background for measurement (e.g., area and volume) and initial understandings of such properties as congruence and symmetry.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Grade 1, Core Content A: Whole Number Relationships **25%**

Students develop an understanding of whole number relationships, including grouping in tens and ones. They compare and order whole numbers to develop an understanding of and solve problems involving the relative magnitude of these numbers. They think of whole numbers between 10 and 100 in groups of tens and ones (especially recognizing the numbers 11 to 19 as 1 group of ten and particular numbers of ones). They understand the sequential order of the counting numbers and represent numbers in a variety of ways.

Learning Goals and Performance (*Locally Assessed) Indicators

1. Understand and explain the relative magnitude of numbers to 100.
   a) *Represent two-digit numbers in a variety of ways and make connections between and among these representations using objects, diagrams, number lines, words, and numerals.
   b) *Read, write, compare and order numbers through 100 by using decades as benchmarks (e.g., 27 is between 20 and 30).
   c) *Group and count objects by 2s, 5s, and 10s.
   d) *Identify numbers missing from a well-defined counting sequence (e.g., 0, 2, 4, __, 8, 10, …)

2. Understand the values of the digits in two-digit numbers.
   a) *Determine the place value (tens, ones) and value of each digit in a number (e.g., the 6 in 63 represents 6 tens or 60 ones).
   b) *Translate between and among different numerical representations of a number (e.g., 63 is 6 tens plus 3 ones or 5 tens plus 13 ones or 4 tens plus 23 ones).
   c) *Identify one more, one less, 10 more, and 10 less than a given number for numbers up to 100.

3. Understand the properties of odd and even numbers.
   a) *Classify a number as odd or even and explain why it is odd or even.

Grade 1, Core Content B: Addition and Subtraction: Operations and Beginning Basic Facts 40%

Students develop an understanding of the operations of addition and subtraction, apply a variety of strategies for basic addition/subtraction facts, and relate addition and subtraction as inverse operations. They use a variety of models to develop an understanding of the meanings of addition and subtraction and strategies to solve arithmetic problems. They understand the connections between counting up and counting back with the operations of addition and subtraction (e.g., adding two is the same as "counting on" two). They use properties of addition (identity, commutative and associative) to add whole numbers, and they create and use increasingly sophisticated strategies based on these properties to solve addition and subtraction problems involving basic facts. By comparing a variety of solution strategies, children relate addition and subtraction as inverse operations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and explain the meaning of addition and subtraction.
   a) *Make connections among a variety of representations, including objects, length-based models (e.g., lengths of connecting cubes), number lines, ten frames, diagrams, words, and number sentences, in order to explain addition and subtraction situations (combining, missing addend, separating, comparing, and relating parts with wholes).
   b) *Explain and justify methods of adding and subtracting numbers on the basis of properties of operations (identity, commutative, associative), place value, and/or the inverse relationship between addition and subtraction.

2. Use a variety of strategies to represent and compute single-digit addition and related subtraction facts.
   a) *Interpret the equal sign as a relational symbol indicating "the same quantity as."
   b) *Use informal strategies to find the unknown quantity in a variety of equations that involve addition and subtraction (e.g., 3 + 4 = __; 7 – __ = 3; __ = 5 – 2).
   c) *Compare solution strategies in order to relate addition and subtraction as inverse operations (e.g., missing addend situations).
   d) *Apply and justify the use of a variety of strategies, including known facts and derived facts (e.g., counting on or counting back, doubling plus one, doubling minus one, making ten), to solve problems.
   e) *Create contextual problems for a variety of mathematical situations (combining, missing addend, separating, comparing, and relating parts with wholes) involving numbers for which any one of the quantities is unknown.
   f) *Solve contextual problems involving a variety of mathematical situations (combining, missing addend, separating, comparing, and relating parts with wholes) for which any one of the quantities is unknown.

Grade 1, Core Content C: Geometric Relationships **20%**

Students develop an understanding of the relationships among geometric shapes. They compose and decompose plane and solid figures (e.g., by putting two congruent isosceles triangles together to make a rhombus), thus building an understanding of part-whole relationships as well as the properties of the original and composite shapes. By combining figures, they recognize them from different perspectives and orientations, describe their geometric attributes and properties, and determine how they are alike and different. Through these processes, they develop a background for measurement (e.g., area and volume) and initial understandings of such properties as congruence and symmetry.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand part-whole relationships and attributes of plane and solid figures.
   a) *Name, create, and sort 2-dimensional shapes, including circles, triangles, rectangles, squares, rhombi, trapezoids, and hexagons.
   b) *Sort 3-dimensional shapes including pyramids, rectangular prisms, cubes, cones, cylinders, and spheres.
   c) *Compose (combine) and decompose (separate) two- and three-dimensional shapes to make other shapes.
   d) *Predict the number of congruent shapes that will cover a region without gaps, then test and verify predictions with models.
   e) *Identify two-dimensional shapes that have line (mirror) symmetry and draw line(s) of symmetry.

GRADE 2

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Grade 2, Core Content A: Base-ten Numeration and Place Value **20%**

Students develop an understanding of the base-ten numeration system and place-value concepts. They count in units and multiples of hundreds. They compare and order numbers. They understand numbers in terms of place value, recognizing that place-value notation is a shorthand for the sums of multiples of powers of 10 (e.g., 853 as 8 hundreds + 5 tens + 3 ones; 800 + 50 + 3).

Grade 2, Core Content B: Addition and Subtraction: Basic Facts Fluency and Multi-digit Computation **35%**

Students demonstrate proficiency with basic facts for addition and subtraction and develop strategies for adding and subtracting multi-digit numbers. They solve problems by applying their understanding of models of addition and subtraction (such as combining or separating sets or using number lines), relationships and properties of number (such as place value), and properties of addition. They apply appropriate methods to estimate sums and differences or calculate them mentally, depending on the context and numbers involved. They demonstrate proficiency (accuracy, efficiency, flexibility, and appropriateness) with computational procedures, including a standard algorithm, for adding and subtracting whole numbers (with sums at least to 1000 and minuends at least to 100). They understand and explain why computational procedures work (on the basis of place value and properties of operations), and use them to solve problems.

Grade 2, Core Content C: Linear Measurement **20%**

Students develop an understanding of linear measurement and facility in measuring lengths. They understand underlying concepts of measurement, such as equal partitioning (the mental activity of slicing the length of an object into equal-sized units) and transitivity (e.g., if object A is longer than object B and object B is longer than object C, then object A is longer than object C). They understand linear measure as an iteration of units (repetition of a single unit with no gaps or overlaps) and use non-standard measurement tools and simple unit rulers (centimeter and inch) on that basis. They understand the need for equal-length units and recognize the inverse relationship between the size of a unit and the number of units used in a particular measurement (e.g., the smaller the unit, the more iterations needed to cover a given length).

Grade 2, Core Content D: Categorical Data **10%**

Students develop an initial understanding of data analysis by formulating questions and simple experiments and by collecting, representing, analyzing, and interpreting data. They pose and investigate questions that can be addressed with categorical data about a small population. They conduct classroom censuses and use a variety of displays to represent and interpret data.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Grade 2, Core Content A: Base-ten Numeration and Place Value **20%**

Students develop an understanding of the base-ten numeration system and place-value concepts. They count in units and multiples of hundreds. They compare and order numbers. They understand numbers in terms of place value, recognizing that place-value notation is a shorthand for the sums of multiples of powers of 10 (e.g., 853 as 8 hundreds + 5 tens + 3 ones; 800 + 50 + 3).

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and explain the relative magnitude of three-digit numbers.
   a) *Make connections among a variety of representations of three-digit whole numbers, including base-ten models, diagrams, words, and numeric forms.
   b) *Read, write, compare and order numbers through 1000 by using decades and hundreds as benchmarks (e.g., 270 is between 200 and 300).

2. Understand and explain the values of the digits in three-digit numbers.
   a) *Recognize the place value (hundreds, tens, ones) and value of each digit in a number (e.g., the 6 in 678 represents 6 hundreds, 60 tens, or 600 ones).
   b) *Translate among different numerical representations of a number (e.g., 678 is 6 hundreds plus 7 tens plus 8 ones or 6 hundreds plus 6 tens plus 18 ones, or 4 hundreds plus 27 tens plus 8 ones).
   c) *Identify 10 more or 10 less and 100 more or 100 less than a given number for numbers to at least 1000.

Grade 2, Core Content B: Addition and Subtraction: Basic Facts Fluency and Multi-digit Computation **35%**

Students demonstrate proficiency with basic facts for addition and subtraction and develop strategies for adding and subtracting multi-digit numbers. They solve problems by applying their understanding of models of addition and subtraction (such as combining or separating sets or using number lines), relationships and properties of number (such as place value), and properties of addition. They apply appropriate methods to estimate sums and differences or calculate them mentally, depending on the context and numbers involved. They demonstrate proficiency (accuracy, efficiency, flexibility, and appropriateness) with computational procedures, including a standard algorithm, for adding and subtracting whole numbers (with sums at least to 1000 and minuends at least to 100). They understand and explain why computational procedures work (on the basis of place value and properties of operations), and use them to solve problems.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and use efficient strategies for computing single-digit addition and subtraction facts.
   a) *Demonstrate proficiency with single-digit addition and related subtraction facts.
   b) *Represent addition and subtraction situations by using objects, diagrams, words, expressions and equations and make connections among the representations.
   c) *Explain and justify addition and subtraction strategies on the basis of place value concepts, properties of operations (identity, commutative, associative) and/or the inverse relationship between addition and subtraction

2. Understand, explain, and use a variety of strategies to proficiently compute multi-digit addition and subtraction problems (sums to 1000 and minuends to 100).
   a) *Analyze a variety of strategies (including a standard algorithm) for addition and subtraction in order to demonstrate their similarities and differences, and to draw conclusions about their efficiency, accuracy, and generalizability.
   b) *Demonstrate proficiency in adding and subtracting multi-digit numbers (sums to 1000 and minuends to 100)
   c) *Estimate sums and differences and/or calculate them mentally depending on the context and numbers involved; use estimates to determine the reasonableness of solutions.
   d) *Recognize and apply the meaning of relational signs (=, ≠, <, >) as distinct from operational signs (+, –) and interpret the meaning of these symbols as true or false in a variety of equations (e.g., 3 + 4 = 7 is true; 3 + 4 = 8 is false.)
   e) *Find the unknown quantity in simple equations that involve addition and subtraction (e.g., 6 + 4 = __; 8 = 14 – __ ; 8 + 4 = __ + 2; __ = 15 – 2).
   f) *Create contextual problems for a variety of mathematical situations (combining, missing addend, separating, comparing, and relating parts with wholes) involving numbers for which any one of the quantities is unknown.
   g) *Solve contextual problems for a variety of mathematical situations (combining, missing addend, separating, comparing, and relating parts with wholes) involving numbers for which any one of the quantities is unknown.

Grade 2, Core Content C: Linear Measurement **20%**

Students develop an understanding of linear measurement and facility in measuring lengths. They understand underlying concepts of measurement, such as equal partitioning (the mental activity of slicing the length of an object into equal-sized units) and transitivity (e.g., if object A is longer than object B and object B is longer than object C, then object A is longer than object C). They understand linear measure as an iteration of units (repetition of a single unit with no gaps or overlaps) and use non-standard measurement tools and simple unit rulers (centimeter and inch) on that basis. They understand the need for equal-length units and recognize the inverse relationship between the size of a unit and the number of units used in a particular measurement (e.g., the smaller the unit, the more iterations needed to cover a given length).

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and explain that the measurement process involves choosing a unit, comparing the unit to the object, and determining the number of units needed to find the length.
   a) *Choose and use nonstandard and standard units (customary and metric) to measure length to the nearest whole unit.
   b) *Estimate length by using nonstandard units and standard units (inches and centimeters).
   c) *Use many copies of the same unit or iterate a single unit to measure the length of an object longer than the unit.
   d) *Realize that when multiple identical units are used for measurement, they must match up with the beginning of the object being measured and there can be no gaps or overlaps between units.
   e) *Use direct comparison and measurement, along with concepts of the transitive property (a>b, b>c, so a>c) when comparing and ordering objects by the attribute of length.
   f) *Describe the inverse relationship between the size of a unit and the number of units to measure an object.
   g) *Recognize each unit on a ruler as completing a linear distance (length) from the zero point, not a location on the tool.

Grade 2, Core Content D: Categorical Data **10%**

Students develop an initial understanding of data analysis by formulating questions and simple experiments and by collecting, representing, analyzing, and interpreting data. They pose and investigate questions that can be addressed with categorical data about a small population. They conduct classroom censuses and use a variety of displays to represent and interpret data.

Learning Goals and Performance (Locally Assessed) Indicators:

1. Understand and explain how to collect, represent, and interpret categorical data in response to questions posed by the class.
   a) *Collect categorical data by conducting classroom censuses based on questions posed by the class.
   b) *Use one-to-one correspondence in constructing frequency tables (with tallies or numbers), bar graphs, and picture graphs in order to represent and interpret the data.
   c) *Make connections among the different representations of the same data and attend to the ideas of distribution and variation within the data set.
   d) *Determine the mode (i.e., the most frequent data point) from a categorical data set (e.g., more students wearing “tennis shoes” than “flip-flops”).
   e) *Compare responses (individual-to-individual and individual-to-group) and acknowledge that results may be different in another class or group.

GRADE 3

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Grade 3, Core Content A: Extending Addition and Subtraction 15%

Students consolidate addition and subtraction strategies for larger numbers. They develop proficiency (accuracy, efficiency, flexibility, and appropriateness) with addition and subtraction of larger numbers, including standard algorithms, understand why these procedures work, and use them to solve problems.

Grade 3, Core Content B: Multiplication and Division: Operations and Beginning Basic Facts 25%

Students develop an initial understanding of multiplication and division, relate multiplication and division as inverse operations, and apply a variety of strategies for multiplication and division basic facts. They understand the meanings of multiplication and division of whole numbers through the use of various representations and situations (equal groups, arrays and area, multiplicative comparisons). They use properties of addition and multiplication to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving basic facts (single-digit factors and related division facts). By comparing a variety of solution strategies, students relate multiplication and division as inverse operations.

Grade 3, Core Content C: Meanings of Fractions and Fractional Relationships 25%

Students develop an understanding of fractions and fraction equivalence by using area, length, and set models. They understand that the size of a fractional part is relative to the size of the whole, and they use fractions to represent numbers that are equal to, less than, or greater than 1. They solve problems that involve comparing and ordering fractions by using models, benchmark fractions, or common numerators or denominators. They understand and use models to identify equivalent fractions.

Grade 3, Core Content D: Properties and Perimeter of Two-dimensional Figures 20%

Students develop an understanding of properties and relationships in two-dimensional space. They use benchmarks for classifying angles. They describe, analyze, compare, and classify two-dimensional shapes by their sides and angles and connect these attributes to the definitions of shapes. Students investigate, describe, and reason about composing and decomposing polygons to make other polygons. Through building, drawing, and analyzing two-dimensional shapes, students understand attributes and properties of two-dimensional space. They use these attributes and properties in solving problems, including applications involving perimeter.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Grade 3, Core Content A: Extending Addition and Subtraction **15%**

Students consolidate addition and subtraction strategies for larger numbers. They develop proficiency (accuracy, efficiency, flexibility, and appropriateness) with addition and subtraction of larger numbers, including standard algorithms, understand why these procedures work, and use them to solve problems.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand, explain, and use a variety of strategies to compute multi-digit addition and subtraction problems proficiently (sums larger than 1000 and minuends larger than 100).
   a) Proficiently compute multi-digit addition and subtraction problems.
   b) *Estimate sums and differences and/or calculate them mentally depending on the context and numbers involved; use estimates to judge the reasonableness of solutions.
   c) *Create single- and multi-step contextual problems for a variety of mathematical situations (combining, missing addend, separating, comparing, and relating parts with wholes) involving multi-digit numbers.
   d) Solve single- and multi-step contextual problems for a variety of mathematical situations (combining, missing addend, separating, comparing, and relating parts with wholes) involving multi-digit numbers.

Grade 3, Core Content B: Multiplication and Division: Operations and Beginning Basic Facts 25%

Students develop an initial understanding of multiplication and division, relate multiplication and division as inverse operations, and apply a variety of strategies for multiplication and division basic facts. They understand the meanings of multiplication and division of whole numbers through the use of various representations and situations (equal groups, arrays and area, multiplicative comparisons). They use properties of addition and multiplication to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving basic facts (single-digit factors and related division facts). By comparing a variety of solution strategies, students relate multiplication and division as inverse operations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand, explain, and apply meanings of multiplication and division.
   a) Represent repeated addition as multiplication and repeated subtraction as division.
   b) Show various representations and situations involving multiplication and division (equal-sized groups, arrays, area models, equal "jumps" on number lines, expressions and equations).
   c) Describe multiplicative relationships in context ("on 1 chair, there are 4 legs, on 2 chairs 8 legs, etc;" "the number of legs is 4 times the number of chairs").
   d) *Use physical models and diagrams to demonstrate differences between partitive (sharing) and quotative (grouping) models of division.
   e) *Compare solution strategies in order to relate multiplication and division as inverse operations.

2. Use and explain a variety of strategies to compute multiplication and division problems (basic facts with factors to 10).
   a) *Compute multiplication basic facts (one-digit factors) and related division facts by using strategies based on properties (identity, commutative, associative, and distributive) and the inverse relationship of multiplication and division.
   b) Use relational signs (=, ≠, <, >) and operational signs (+, –, ×, ÷) to represent multiplication and division basic facts.
   c) *Create contextual problems for a variety of mathematical situations (addition, subtraction, one-digit factor multiplication and/or division [partitive and quotative]).
   d) Solve contextual problems for a variety of mathematical situations (addition, subtraction, one-digit factor multiplication and/or division [partitive and quotative]).

Grade 3, Core Content C: Meanings of Fractions and Fractional Relationships 25%

Students develop an understanding of fractions and fraction equivalence by using area, length, and set models. They understand that the size of a fractional part is relative to the size of the whole, and they use fractions to represent numbers that are equal to, less than, or greater than 1. They solve problems that involve comparing and ordering fractions by using models, benchmark fractions, or common numerators or denominators. They understand and use models to identify equivalent fractions.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand concepts of fractions equal to, less than, or greater than 1.
   a) Relate different representations of the same fraction (e.g., diagram, position on a number line, words, and numerical representation).
   b) Define regions, lines, and sets of objects as a whole, and divide the whole into equal parts; represent fractions with these models.
   c) Identify and apply the meaning of the denominator of a fraction as the number of equal parts of the unit whole and the numerator of a fraction as the number of equal parts being considered.
   d) *Given a fraction, and its representation as a part of a whole, construct a whole.

2. Understand and use various methods to model and compare fractions and to identify equivalent fractions.
   a) Compare and order common fractions in a variety of ways such as using physical models of fractions, relating fractions to benchmarks (e.g., closer to 0, 1/2, or 1), and comparing fractions with like numerators or denominators.
   b) Use models, including the number line, to identify equivalent fractions.

Grade 3, Core Content D: Properties and Perimeter of Two-dimensional Figures 20%

Students develop an understanding of properties and relationships in two-dimensional space. They use benchmarks for classifying angles. They describe, analyze, compare, and classify two-dimensional shapes by their sides and angles and connect these attributes to the definitions of shapes. Students investigate, describe, and reason about composing and decomposing polygons to make other polygons. Through building, drawing, and analyzing two-dimensional shapes, students understand attributes and properties of two-dimensional space. They use these attributes and properties in solving problems, including applications involving perimeter.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand, explain, and use concepts of angle and angle measurement.
   a) *Describe and interpret angles in various contexts, including static angles in geometric figures and dynamic angles involving motion (e.g., hands of a clock, the opening of a door).
   b) *Develop references or benchmarks (45º, 90º, 180º, and 270º) for angle measures by using physical models such as pattern blocks and tangrams.
   c) Classify angles as right, acute, or obtuse.

2. Understand that two-dimensional shapes can be created, described, compared, and classified on the basis of their attributes and properties.
   a) Identify and describe attributes classifying triangles (e.g., two equal sides for the isosceles triangle, right angle for the right triangle).
   b) Identify and describe attributes classifying quadrilaterals (e.g., parallel sides for the parallelogram, right angles or perpendicular sides for the rectangle).
   c) Identify right angles in geometric figures and determine whether other angles are greater than (obtuse) or less than (acute) a right angle.
   d) *Predict the results of putting together and taking apart two-dimensional shapes, then test predictions with models.
   e) *Create shapes satisfying particular specifications such as number of sides, relative length of sides, or relationships between sides (e.g., triangle with no congruent sides, pentagon with two parallel sides).

3. Understand, explain, and apply concepts of perimeter and perimeter measurement.
   a) Recognize that any point on a ruler can be used as a starting point for measurement, as long as adjustments are made for non-zero starting points.
   b) *Use customary and metric units to estimate and measure the perimeter of real objects.
   c) Use customary and metric units to determine the perimeter of polygons.
   d) Create a variety of rectangles with a given perimeter.
   e) Use perimeter concepts and knowledge of metric and customary measurement systems to solve problems involving rectangles, including the measure of an unknown side.

GRADE 4

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Grade 4, Core Content A: Multiplication and Division: Basic Facts Fluency and Multi-digit Computation **30%**

Students develop proficiency with multiplication and division basic facts, multi-digit multiplication, and division with single-digit divisors. They apply their understanding of models for multiplication and division, place value of numbers, and properties of operations (in particular, the distributive property) as they multiply and divide whole numbers. They select appropriate methods and apply them accurately to estimate products or calculate them mentally, depending on the context and numbers involved. They develop proficiency (accuracy, efficiency, flexibility, and appropriateness) with computational procedures, including a standard algorithm, for multiplying and dividing whole numbers, understand why these procedures work and use them to solve problems. In preparation for and as a result of working with larger numbers in multiplication and division, students also expand their understanding of place value.

Grade 4, Core Content B: Decimal Place Value and Connections to Fractions and Percents **30%**

Students develop an understanding of decimals, including the connections among fractions, decimals, and percents. They understand decimal notation as an extension of the base-ten system of writing whole numbers that is useful for representing more numbers, including numbers between 0 and 1, between 1 and 2, and so on. Students relate their understanding of fractions to reading and writing decimals that are greater than or less than 1, identifying equivalent decimals, comparing and ordering decimals, and estimating decimal or fractional amounts in problem solving. They relate benchmark percents with decimals and connect equivalent fractions with decimals by comparing models to symbols and locating equivalent symbols on the number line.

Grade 4, Core Content C: Area Measurement and Transformations **15%**

Students develop a basic understanding of area and transformations. They recognize area as an attribute of two-dimensional regions and learn that they can quantify area by finding the total number of same-sized units of area that cover the shape without gaps or overlaps. They understand that a plane-covering unit is needed to measure area and that a square which is 1 unit on a side is the standard unit for this measurement. Students connect area measure to the area model that they have used to represent multiplication, and they use this connection to justify the formula for the area of a rectangle. Students transform two-dimensional shapes to check for congruence and symmetry.

Grade 4, Core Content D: Data Organization and Analysis **10%**

Students develop an understanding of data analysis by formulating questions, collecting or using available data, analyzing tools of data representation, and using measures of center and spread to interpret data. They formulate and investigate questions that can be addressed with numerical data. They conduct simple experiments and use a variety of displays to represent and interpret data. Students attend to the distribution of data by describing its center and spread, and recognize limitations of the scope of inference beyond the experiment.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Grade 4, Core Content A: Multiplication and Division Basic Facts and Multi-digit Computation 30%

Students develop proficiency with basic facts for multiplication and division and multi-digit multiplication and division with single-digit divisors. They apply their understanding of models for multiplication and division, place value of numbers, and properties of operations (in particular, the distributive property) as they multiply and divide whole numbers. They select appropriate methods and apply them accurately to estimate products or calculate them mentally, depending on the context and numbers involved. They develop proficiency (accuracy, efficiency, flexibility, and appropriateness) with computational procedures, including a standard algorithm, for multiplying and dividing whole numbers, understand why these procedures work and use them to solve problems. In preparation for and as a result of working with larger numbers in multiplication and division, students also expand their understanding of place value.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and use efficient strategies for computing multiplication and division facts and use equations to represent the computations.
   a) *Demonstrate proficiency with multiplication and division basic facts (single-digit factors and related division facts).
   b) Recognize and use relational (=,≠, <, >) and operational (+, –, x, ÷) symbols to represent mathematical equations using multiplication and division.
   c) Find the unknown quantity in a variety of simple equations (e.g., 20 x 4 = ___; 6 = 48 ÷ ___; 4 x 3 = ___ x 2; ___ = 7 x 8) that involve addition, subtraction, multiplication and/or division.

2. Extend understanding of place value concepts and the operations of multiplication and division.
   a) Determine the place value (hundred thousands through ones) and the value of each digit in a number (e.g., the 6 in 678,000 represents 6 hundred thousands, 60 ten thousands or 600 thousands) and translate among these representations.
   b) Make connections among representations of multi-digit multiplication situations with objects, diagrams (especially area models), words, expressions, and equations.
   c) Describe the effects of multiplying whole numbers by 10, 100, or 1000.
   d) *Explain what a remainder represents in a contextual problem involving division.
   e) *Analyze a variety of strategies (including a standard algorithm) for multiplication and division in order to demonstrate their similarities and differences, and to draw conclusions about their efficiency, accuracy, and generalizability.
   f) *Explain and justify multi-digit multiplication and division strategies on the basis of place value and properties of operations (identity, zero, commutative, associative, and distributive).

3. Understand, explain, and use efficient strategies to compute multiplication problems (with products to 10,000) and division problems (with up to three-digit dividend and one-digit divisor).
   a) Use efficient strategies, including a standard algorithm, to solve multiplication (multi-digit) and division (one-digit divisor and up to three-digit dividend) problems proficiently.
   b) *Estimate products and/or calculate them mentally depending on the context and numbers involved; use estimates to judge the reasonableness of solutions.
   c) Create single- and multi-step contextual problems for a variety of mathematical situations (multi-digit addition, multi-digit subtraction, multi-digit multiplication and/or division with one-digit divisor [partitive and quotative models]).
   d) Solve single- and multi-step contextual problems (multi-digit addition, multi-digit subtraction, multi-digit multiplication and/or division with one-digit divisor [partitive and quotative models]).

Grade 4, Core Content B: Decimal Place Value and Connections to Fractions and Percents 30%

Students develop an understanding of decimals, including the connections among fractions, decimals, and percents. They understand decimal notation as an extension of the base-ten system system of writing whole numbers that is useful for representing more numbers, including numbers between 0 and 1, between 1 and 2, and so on. Students relate their understanding of fractions to reading and writing decimals that are greater than or less than 1, identifying equivalent decimals, comparing and ordering decimals, and estimating decimal or fractional amounts in problem solving. They relate benchmark percents with decimals and connect equivalent fractions with decimals by comparing models to symbols and locating equivalent symbols on the number line.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and apply the meaning of decimals and the relationships among the place values of their digits.
   a) Make connections between and among a variety of representations of hundredths and tenths, including base ten models, meter sticks, words, standard and expanded forms.
   b) Recognize and apply the concepts underlying place value (tenths, hundredths) by identifying the value of each digit in a decimal (e.g., the 7 in 0.78 represents 7 tenths or 70 hundredths) and translate among these representations.
   c) Identify a tenth more or a tenth less, and a hundredth more or a hundredth less, than a given number.

2. Understand and explain relationships among commonly used fractions, decimals, and percents.
   a) Model fractions (halves, fourths, eighths, fifths and tenths) on a 10 x 10 grid and use this representation to convert fractions to decimals.
   b) Relate fractions with denominators of ten and one hundred to equivalent decimals.
   c) Relate benchmark percents (0%, 25%, 50%, 75%, and 100%) to fractions with denominators of 100 and to decimals in the hundredths.
   d) Identify equivalent fractions, decimals (less than one, equivalent to one, and greater than one) and percents, with and without models, including locations on a number line.
   e) Use a variety of methods to compare and order fractions, decimals, and percents.

Grade 4, Core Content C: Area Measurement and Transformations **15%**

Students develop a basic understanding of area and transformations. They recognize area as an attribute of two-dimensional regions and learn that they can quantify area by finding the total number of same-sized units of area that cover the shape without gaps or overlaps. They understand that a plane-covering unit is needed to measure area and that a square which is 1 unit on a side is the standard unit for this measurement. Students connect area measure to the area model that they have used to represent multiplication, and they use this connection to justify the formula for the area of a rectangle. Students transform two-dimensional shapes to check for congruence and symmetry.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand, explain, and apply the concepts of area and area measurement as related to rectangles.
   a) Find the area of a variety of two-dimensional figures by using physical models (e.g., square units to cover a shape with no gaps or overlaps or a transparent grid placed over a shape) to count the total number of units.
   b) Derive and use the area formula for a rectangle ($b \cdot h$) and connect it with the area model for multiplication.
   c) Given a linear or area measurement situation, select the appropriate type (one dimensional or two dimensional) and size (i.e. inches/feet, centimeters/meters, etc.) of unit.
   d) Demonstrate that rectangles with the same area can have different perimeters, and that rectangles with the same perimeter can have different areas.
   e) Use area and perimeter concepts and knowledge of metric and customary measurement systems to solve problems involving rectangles including the measure of unknown sides.

2. Understand and explain the meanings and uses of transformations.
   a) Use translations (slides), reflections (flips), and rotations (turns) to transform two-dimensional shapes.
   b) Predict the result of a transformation.
   c) Determine whether two shapes are congruent by using a combination of translations, reflections, and/or rotations with models.
   d) Use transformations to determine whether a two-dimensional shape has line and/or rotational symmetry.

Grade 4, Core Content D: Data Organization and Analysis **10%**

Students develop an understanding of data analysis by formulating questions, collecting or using available data, analyzing tools of data representation, and using measures of center and spread to interpret data. They formulate and investigate questions that can be addressed with numerical data. They conduct simple experiments and use a variety of displays to represent and interpret data. Students attend to the distribution of data by describing its center and spread, and recognize limitations of the scope of inference beyond the experiment.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand how to formulate and answer questions that can be addressed by conducting simple experiments and collecting numerical data.
   a) *Formulate questions that can be addressed with data.
   b) Collect or use available numerical data in order to draw conclusions and answer questions.

2. Understand the tools for exploring distributions (including measures of center and spread), the appropriateness of data displays, and limitations of inference.
   a) Represent distributions of data by using a variety of displays, including tables, bar graphs, line graphs, line plots, and discuss the appropriateness of each type of display.
   b) Describe the distribution of data in terms of its center (mode and median) and spread (range).
   c) Compare related data sets on the basis of measures of center (mode and median) and spread (range).
   d) Recognize limitations in the scope of inference beyond the experiment.

GRADE 5

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Grade 5, Core Concept A: Division of Whole Numbers 30%

Students extend their understanding of and proficiency with whole number division and explain why strategies work on the basis of place value and properties of operations. They apply their understanding of place value, properties, models for division, and the relationship of division to multiplication as they find quotients involving multi-digit dividends. They select appropriate methods and apply them accurately to estimate quotients or calculate them mentally, depending on the context and numbers involved. They develop proficiency (accuracy, efficiency, flexibility, and appropriateness) with computational procedures, including a standard algorithm, for dividing whole numbers and they use these procedures to solve problems. They consider the context in which a problem is situated to select the most useful form of the quotient for the solution, and they interpret it appropriately.

Grade 5, Core Content B: Addition and Subtraction of Fractions and Decimals 30%

Students develop an understanding of and proficiency with addition and subtraction strategies for fractions (including mixed numbers) and decimals. They apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with like and unlike denominators, and their understanding of decimal models, place value, and properties to add and subtract decimals. They use appropriate methods to estimate sums and differences and consider estimates in judging the reasonableness of their solutions. They develop proficiency (accuracy, efficiency, flexibility, and appropriateness) with computational strategies (including standard algorithms) for adding and subtracting fractions and decimals. They explain why procedures work on the basis of place value and properties of operations. They add and subtract fractions and decimals to solve problems, including problems involving measurement.

Grade 5, Core Content C: Properties of Three-dimensional Shapes, Volume and Surface Area 15%

Students develop an understanding of the properties of three-dimensional shapes, including volume and surface area. They relate two-dimensional shapes to three-dimensional shapes and analyze properties of polyhedral solids, describing them by the number of edges, faces, or vertices as well as the types of faces. Students recognize volume as an attribute of three-dimensional space. They understand that they can quantify volume by finding the total number of same-sized units of volume that fill the space without gaps or overlaps. They understand that a space-filling unit is needed to measure volume and that a cube which is 1 unit on an edge is the standard unit for this measurement. They select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume. They decompose three-dimensional shapes and find surface areas and volumes of rectangular prisms.

Grade 5, Core Content D: Basic Probability Concepts 10%

Students develop an understanding of probability through the contexts of simple experiments and their outcomes. They begin to describe events as likely by using such words as *certain, equally likely,* and *impossible*. They learn that probability is a measurement of the likelihood of events and quantify the probability of an event as a number between 0 and 1. Moreover, they learn that sample space is the set of all possible outcomes and that the sum of the probabilities of all sample space outcomes is 1. They begin to quantify likelihood by conducting experiments that have only a few outcomes. Through these experiences, students encounter the idea that although they cannot determine an individual outcome, they can predict the frequency of various outcomes.

The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Grade 5, Core Concept A: Division of Whole Numbers **30%**

Students extend their understanding of and proficiency with whole number division and explain why strategies work on the basis of place value and properties of operations. They apply their understanding of place value, properties, models for division, and the relationship of division to multiplication as they find quotients involving multi-digit dividends. They select appropriate methods and apply them accurately to estimate quotients or calculate them mentally, depending on the context and numbers involved. They develop proficiency (accuracy, efficiency, flexibility, and appropriateness) with computational procedures, including a standard algorithm, for dividing whole numbers and they use these procedures to solve problems. They consider the context in which a problem is situated to select the most useful form of the quotient for the solution, and they interpret it appropriately.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand multi-digit division (with divisors to 100 and dividends to 10,000).
   a) Make connections among representations of multi-digit division situations with objects, diagrams, words, expressions, and equations.
   b) Describe the effect of dividing by 10 or 100 on the quotient.
   c) *Analyze a variety of strategies (including a standard algorithm) for division in order to demonstrate their similarities and differences, and to draw conclusions about their efficiency, accuracy, and generalizability.
   d) *Explain and justify multi-digit division strategies on the basis of place value, relationships among operations, and properties of operations (identity, distributive).

2. Understand, explain, and use a variety of strategies to compute division problems proficiently (divisors to 100 and dividends to 10,000).
   a) Apply and explain efficient strategies, including a standard algorithm, to divide with two-digit divisors proficiently.
   b) Express remainders as whole numbers or fractions and interpret remainders appropriately in contextual problems.
   c) *Estimate products and quotients and/or calculate them mentally depending on the context and numbers involved; use estimates to judge the reasonableness of solutions.
   d) Recognize and apply the meaning of relational signs (=, ≠, <, >) as distinct from operational signs (+, –, x, ÷) and interpret the meaning of these symbols as true or false in a variety of equations and inequalities (e.g., 12 ÷ 4 = 3 is true; 12 ÷ 4 ≠ 3 is false).
   e) Find the unknown quantity in a variety of simple equations (16 x 4 = ___; 24 = 48 ÷ ___; ___ = 7 x 21; 4 x 13 = ___ x 2) that involve operations with whole numbers.
   f) *Create single- and multi-step contextual problems involving addition, subtraction, multiplication and/or division of whole numbers.
   g) Solve single- and multi-step contextual problems involving addition, subtraction, multiplication and/or division of whole numbers.

Grade 5, Core Content B: Addition and Subtraction of Fractions and Decimals 30%

Students develop an understanding of and proficiency with addition and subtraction strategies for fractions (including mixed numbers) and decimals. They apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with like and unlike denominators, and their understanding of decimal models, place value, and properties to add and subtract decimals. They use appropriate methods to estimate sums and differences and consider estimates in judging the reasonableness of their solutions. They develop proficiency (accuracy, efficiency, flexibility, and appropriateness) with computational strategies (including standard algorithms) for adding and subtracting fractions and decimals. They explain why procedures work on the basis of place value and properties of operations. They add and subtract fractions and decimals to solve problems, including problems involving measurement.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand, explain, and apply strategies to add and subtract fractions proficiently (including mixed numbers with like and unlike denominators).
   a) Make connections among representations of fraction addition and subtraction situations with objects, diagrams, words, expressions and equations.
   b) Apply concepts of common multiples, common factors, prime and composite numbers as needed to support the addition and subtraction of fractions.
   c) *Analyze a variety of strategies, including standard algorithms, for addition and subtraction of fractions in order to demonstrate their similarities and differences, and to draw conclusions about their efficiency, accuracy, and genralizability.
   d) Apply and explain efficient strategies, including standard algorithms, to add and subtract fractions, including mixed numbers, proficiently.
   e) *Estimate fraction sums and differences and/or calculate them mentally, depending on the context and numbers involved; use estimates to judge the reasonableness of solutions.
   f) *Create contextual problems for a variety of mathematical situations (combining, missing addend, separating, comparing, and relating parts with wholes) involving fractions for which any one of the quantities is unknown.
   g) Proficiently solve contextual problems for a variety of mathematical situations (combining, missing addend, separating, comparing, and relating parts with wholes) involving fractions for which any one of the quantities is unknown.

2. Understand, explain, and apply strategies to add and subtract decimals proficiently.
   a) Make connections among representations of decimal addition and subtraction situations with objects, diagrams, words, expressions and equations.
   b) *Relate the addition and subtraction of decimals to the addition and subtraction of fractions.
   c) *Analyze a variety of strategies, including standard algorithms, for addition and subtraction of decimals in order to demonstrate their similarities and differences, and to draw conclusions about their efficiency, accuracy, and generalizability.
   d) Apply and explain efficient strategies, including standard algorithms, to add and subtract decimals proficiently.
   e) *Estimate decimal sums and differences and/or calculate them mentally, depending on the context and numbers involved; use estimates to judge the reasonableness of solutions.
   f) *Create contextual problems for a variety of mathematical situations (combining, missing addend, separating, comparing, and relating parts with wholes) involving decimal numbers for which any one of the quantities is unknown.
   g) Proficiently solve contextual problems for a variety of mathematical situations (combining, missing addend, separating, comparing, and relating parts with wholes) involving decimal numbers for which any one of the quantities is unknown.

Grade 5, Core Content C: Properties of Three-dimensional Shapes, Volume and Surface Area 15%

Students develop an understanding of the properties of three-dimensional shapes, including volume and surface area. They relate two-dimensional shapes to three-dimensional shapes and analyze properties of polyhedral solids, describing them by the number of edges, faces, or vertices as well as the types of faces. Students recognize volume as an attribute of three-dimensional space. They understand that they can quantify volume by finding the total number of same-sized units of volume that fill the space without gaps or overlaps. They understand that a space-filling unit is needed to measure volume and that a cube which is 1 unit on an edge is the standard unit for this measurement. They select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume. They decompose three-dimensional shapes and find surface areas and volumes of rectangular prisms.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand the properties of three-dimensional shapes.
   a) Determine the number of edges, faces, and vertices of a given polyhedron.
   b) Identify the types of faces of a given polyhedron.
   c) *Compose larger polyhedra from smaller ones; decompose larger polyhedra into smaller ones.
   d) Analyze and compare three-dimensional shapes on the basis of their edges, faces, and vertices.
   e) Relate rectangular prisms to the two-dimensional shapes (nets) from which they can be created.

2. Understand and explain the concepts of surface area and volume as they relate to rectangular prisms.
   a) Use area concepts to solve problems involving surface areas (areas of nets) of rectangular prisms.
   b) *Find the volume of rectangular prisms by using cubic units to fill them, with no gaps or overlaps, then counting the total number of units.
   c) *Identify, organize, and use the underlying structure of cubes filling a rectangular prism (a series of layers) to find the volume of rectangular prisms.
   d) *Solve problems that involve estimating or physically measuring the volume of rectangular prisms.
   e) Distinguish among appropriate units for linear, area, and volume measurement situations.

Grade 5, Core Content D: Basic Probability Concepts **10%**

Students develop an understanding of probability through the contexts of simple experiments and their outcomes. They begin to describe events as likely by using such words as *certain, equally likely,* and *impossible*. They learn that probability is a measurement of the likelihood of events and quantify the probability of an event as a number between 0 and 1. Moreover, they learn that sample space is the set of all possible outcomes and that the sum of the probabilities of all sample space outcomes is 1. They begin to quantify likelihood by conducting experiments that have only a few outcomes. Through these experiences, students encounter the idea that although they cannot determine an individual outcome, they can predict the frequency of various outcomes.

Learning Goals and Performance (*Locally Assessed) Indicators

1. Understand and explain that probability is a measurement of the likelihood of events and that the probability of an event is based on the set of all possible events called the sample space.
   a) List all possible outcomes for simple experiments (e.g., predicting sums when rolling two number cubes).
   b) Recognize whether an outcome of an experiment or simulation is *impossible, unlikely, possible, likely,* or *certain*, and whether two or more events are *equally likely.*
   c) Represent the probability of an event, which ranges from 0 (*impossible*) to 1 (*certain*), with a fraction, decimal, or percent.
   d) *Predict the likelihood of an outcome prior to an experiment and compare the predicted probability with the experimental results.

Core Content, Learning Goals and Performance Indicators

Grades 6-8

GRADE 6

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Grade 6, Core Content A: Multiplication and Division of Fractions and Decimals 20%

Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. They use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain procedures for multiplying and dividing decimals. Students use common algorithms to multiply and divide fractions and decimals efficiently and accurately. They multiply and divide fractions and decimals to solve problems, including multistep problems and problems involving measurement.

Grade 6, Core Content B: Ratios, Rates and Percents 15%

Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., "If 5 items cost $3.75 and all items are the same price, then I can find the cost of 12 items by first dividing $3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12"). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. They expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students view percents as ratios and represent percents visually and numerically. They move flexibly between and among the percent, fraction, and decimal representations of numbers and use these representations in solving a variety of problems.

Grade 6, Core Content C: Extending Properties and Measures of Two-dimensional Figures 20%

Students identify and classify triangles and quadrilaterals by their properties including angle measure, side lengths, parallelism, perpendicularity, and symmetry. They develop formulas for the areas of these figures, reinforcing the connection between algebra and geometry. Students extend what they know about area and perimeter to more complex two-dimensional figures including circles. Students use what they learn about triangles, quadrilaterals and circles to solve a variety of problems.

Grade 6, Core Content D: Mathematical Expressions and Equations 15%

Students write mathematical expressions and equations that correspond to given situations, they evaluate expressions, and they use expressions and equations to solve problems. They understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately. Students understand that expressions in different forms can be equivalent, and they use properties to rewrite an expression to represent a quantity in a different way. Students know that the solutions to an equation are the values of the variables that make the equation true. They solve simple one-step equations by using number sense, properties of operations, and the idea of maintaining equality on both sides of an equation.

Grade 6, Core Concept E: Sample Surveys and Data Distributions 15%

Students formulate and answer questions by collecting and analyzing univariate numerical data, exploring methods of random selection, and recognizing the distinction among a population, a census, and a sample. They summarize and compare two or more distributions using a variety of displays and numerical summaries and describe differences between them with respect to center, spread, and shape, and recognize limitations in the scope of inference beyond the experiment.

Grade 6, Core Content A: Multiplication and Division of Fractions and Decimals 20%

Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. They use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain procedures for multiplying and dividing decimals. Students use common algorithms to multiply and divide fractions and decimals efficiently and accurately. They multiply and divide fractions and decimals to solve problems, including multi-step problems and problems involving measurement.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Develop proficiency in multiplying and dividing fractions and decimals.
   a) *Estimate the results of computations with fractions and decimals and judge the reasonableness of the results.
   b) Multiply and divide whole numbers and decimals by 1000, 100, 10, 1, 0.1, 0.01, and 0.001.
   c) Explain the relationship between multiplication and division and justify procedures for multiplying and dividing fractions and decimals.
   d) Multiply and divide fractions and decimals proficiently.
   e) *Describe the effect of multiplying or dividing by a number between zero and one, by one, and by a number greater than one.
   f) Solve single- and multi-step word problems involving multiplication and division of fractions and decimals and verify the solutions.

Grade 6, Core Content B: Ratios, Rates and Percents **15%**

Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., "If 5 items cost $3.75 and all items are the same price, then I can find the cost of 12 items by first dividing $3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12"). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. They expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students view percents as ratios and represent percents visually and numerically. They move flexibly between and among the percent, fraction, and decimal representations of numbers and use these representations in solving a variety of problems.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Understand and use ratios to represent quantitative relationships.
   a) Identify and write ratios as comparisons of part-to-part and part-to-whole relationships using appropriate notation to describe problem situations.
   b) *Justify why two different pairs of numbers may be used to represent the same ratio.
   c) Solve single- and multi-step word problems involving ratios and rates.
   d) Represent and model ratios associated with whole-number percents that are less than or equal to 100%.

2. Demonstrate flexibility with fractions, decimals, and percents.
   a) Move flexibly between and among the fractional, decimal, and percent representations of a number in mathematical and contextual situations.
   b) Compare and order fractions, decimals, and percents and find their approximate locations on a number line.

Grade 6, Core Content C: Extending Properties and Measures of Two-dimensional Figures 20%

Students identify and classify triangles and quadrilaterals by their properties including angle measure, side lengths, parallelism, perpendicularity, and symmetry. They develop formulas for the areas of these figures, reinforcing the connection between algebra and geometry. Students extend what they know about area and perimeter to more complex two-dimensional figures including circles. Students use what they learn about triangles, quadrilaterals and circles to solve a variety of problems.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Represent, identify and classify geometric figures from written or verbal descriptions, measurements, and properties using sketches, figures represented on the coordinate plane, grids, or models.
   a) Identify and use properties (including congruency, parallelism, perpendicularity, and symmetry) to classify quadrilaterals and triangles.
   b) Draw or create two-dimensional figures or models with specified measures and properties including the use of first-quadrant coordinates.
   c.) Measure angles using a protractor and other appropriate tools.

2. Develop and apply formulas for perimeter and area of triangles, quadrilaterals with at least one pair of parallel sides, circles, and composite figures made from these shapes.
   a) *Describe the relationship between the circumference and diameter of a circle (circumference = π x diameter) and use this relationship to develop general formulas such as $C = 2\pi r$ *or* $C = \pi d$ and $A = \pi r^2$.
   b) *Describe relationships between the areas of various polygons (e.g., parallelogram and rectangle or triangle and parallelogram with the same base and height) including how these relationships are expressed in the formulas for the figures.
   c) Determine the area of quadrilaterals and triangles using appropriate units of measure and solve single- and multi-step word problems involving measures of these figures.
   d) Determine the circumference and area of circles using appropriate units of measure and solve single- and multi-step word problems involving the relationships among the radius, diameter, circumference, and area of circles.

Grade 6, Core Content D: Polynomial Expressions and Equations **15%**

Students write polynomial expressions and equations that correspond to given situations, they evaluate and simplify expressions, and they use expressions and equations to solve problems. They understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately. Students understand that expressions in different forms can be equivalent, and they use properties to rewrite an expression to represent a quantity in a different way. Students know that the solutions to an equation are the values of the variables that make the equation true. They solve simple one-step equations by using number sense, properties of operations, and the idea of maintaining equality on both sides of an equation.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Write polynomial expressions and equations that correspond to a given situation; evaluate and simplify expressions.
   a) Write mathematical expressions and equations with variables to represent a given situation.
   b) Simplify expressions using properties (associative, commutative, distributive) and operation
   c) Evaluate mathematical expressions (using substitution when variables are involved) applying the commutative, associative, and distributive properties and order of operations.
   d) *Solve simple equations generated from representing situations mathematically using informal strategies (e.g., guess and check, working backwards).

Grade 6, Core Concept E: Sample Surveys and Data Distributions **15%**

Students formulate and answer questions by collecting and analyzing univariate numerical data, exploring methods of random selection, and recognizing the distinction among a population, a census, and a sample. They summarize and compare two or more distributions using a variety of displays and numerical summaries and describe differences between them with respect to center, spread, and shape, and recognize limitations in the scope of inference beyond the experiment.

Learning Goals/Performance (*Locally Assessed) Indicators:

1. Formulate questions, identify the numerical attributes on which to collect data, decide how to measure the attribute, determine and implement a data collection process.
   a) *Design, conduct and evaluate sample surveys.
   b) Distinguish between a *population* and *sample*, identify methods of sample selection and sources of bias associated with non-random selection.

2. Compare two or more data distributions using displays and numerical summaries and describe differences between them with respect to center, spread, and shape. Recognize limitations in the scope of inference beyond the experiment.
   a) Summarize and compare distributions using numerical summaries and data displays, including relative frequency tables, box plots, circle graphs (pie charts), back-to-back stem-and-leaf plots, double bar graphs, and histograms.
   b) Quantify measures of center (mean, median, and mode), interpret the meaning of these measures in context, explain the influences of outliers on each measure, and justify which statistic is more appropriate for summarizing a given data set.
   c) *Describe how mean, median, mode or range relates to the shape of the distribution.
   d) *Distinguish between interpretations of the mean as the “fair share” value for data and as the “balancing point” of the corresponding data distribution.

GRADE 7

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Grade 7, Core Content A: Integers and Linear Equations **30%**

Students extend understandings of addition, subtraction, multiplication, and division, together with their properties, to all rational numbers, including negative integers. By applying properties of arithmetic and considering negative numbers in everyday contexts students explain why the rules for adding, subtracting, multiplying, and dividing with negative numbers make sense. Students use the arithmetic of rational numbers as they formulate and solve linear equations in one variable and use these equations to solve problems. Students make strategic choices of procedures to solve linear equations in one variable and implement them efficiently, understanding that when they use the properties of equality to express an equation in a new way, solutions that they obtain for the new equation also solve the original equation.

Grade 7, Core Content B: Proportionality and Similarity **25%**

Students extend their work with ratios to develop an understanding of proportionality that they apply to solve single and multi-step problems in numerous contexts. They use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease. They also solve problems about similar objects (including figures) by using scale factors that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and identify the unit rate as the slope of the related line. They distinguish proportional relationships from other relationships.

Grade 7, Core Content C: Surface Area and Volume **20%**

By decomposing two- and three-dimensional shapes into smaller, component shapes, students find surface areas and develop and justify formulas for the surface areas and volumes of prisms and cylinders. As students decompose prisms and cylinders by slicing them, they develop and understand formulas for their volumes (Volume = Area of base × Height). They apply these formulas in problem solving to determine volumes of prisms and cylinders. Students investigate how surface area and volume are affected when one or more dimensions of a figure are changed. They select appropriate two- and three-dimensional shapes to model real-world situations, create two-dimensional drawings of three-dimensional figures, and solve a variety of problems (including multi-step problems) involving surface areas and volumes of prisms and cylinders.

Grade 7, Core Concept D: Experimental and Theoretical Probability **10%**

Students conduct one-stage experiments, use experimental data to estimate probabilities, and, where possible, compare experimental and theoretical probabilities and examine experimental probability in the long run. Students recognize that repetitions of an experiment may result in different outcomes and describe the variation in outcomes produced in an experiment. Students recognize that small samples are often not representative of the population from which they are drawn, and with the collection of more data the experimental probability of a particular outcome approaches the theoretical probability.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Grade 7, Core Content A: Integers and Linear Equations **30%**

Students extend understandings of addition, subtraction, multiplication, and division, together with their properties, to all rational numbers, including negative integers. By applying properties of arithmetic and considering negative numbers in everyday contexts students explain why the rules for adding, subtracting, multiplying, and dividing with negative numbers make sense. Students use the arithmetic of rational numbers as they formulate and solve linear equations in one variable and use these equations to solve problems. Students make strategic choices of procedures to solve linear equations in one variable and implement them efficiently, understanding that when they use the properties of equality to express an equation in a new way, solutions that they obtain for the new equation also solve the original equation.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Represent, order, and compare integers.
   a) *Explain everyday contexts (e.g., owing money, measuring elevations above and below sea level) where integers are used to quantify situations.
   b) Compare (<, >, =) and order integers, locate integers on a number line, recognize the absolute value as an integer's distance from zero on a number line and apply to problem situations.

2. Model operations, compute fluently and solve problems with integers.
   a) *Show how operations on integers can be modeled and use the models to develop and explain efficient procedures for computing with integers.
   b) Compute fluently with integers in problem situations, applying order of operations and the absolute value of integers.
   c) *Estimate and judge the reasonableness of results involving integer operations.
   d) Identify and use properties (closure, associative, commutative, identity, inverse, zero) in computing with integers as well as order of operations.
   e) Use exponents to represent repeated multiplication and calculate the value of expressions represented with exponential notation.

3. Represent situations and solve problems that involve linear relationships.
   a) Represent linear relationships with equations using both explicit and recursive (Next, Now) notation.
   b) Solve one- and two-step linear equations with integer coefficients.
   c) Write and solve one- or two-step linear equations that correspond to problem situations.
   d) Represent linear relationships using graphs, tables, and verbal descriptions.
   e) *Identify relationships as linear or nonlinear and contrast their properties (e.g., rate of change) from tables, graphs, or equations.

Grade 7, Core Content B: Proportionality and Similarity **25%**

Students extend their work with ratios to develop an understanding of proportionality that they apply to solve single and multi-step problems in numerous contexts. They use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease. They also solve problems about similar objects (including figures) by using scale factors that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and identify the unit rate as the slope of the related line. They distinguish proportional relationships from other relationships.

Learning Goals and Performance(*Locally Assessed) Indicators:

1. Develop computational fluency in working with ratios, percents, and proportional situations and apply this fluency to estimate the solution to and solve a variety of problems.
   a) Use proportionality to model and solve problems, including percent applications and measurement conversions.
   b) *Estimate solutions to percent problems.
   c) Use proportionality to interpret circle graphs.

2. Identify, describe, and apply similarity relationships to find measures of corresponding parts in similar figures and apply scales/scale factors to measurements in drawings and maps.
   a) Given similar two-dimensional figures, identify the scale factor and describe the relationships between the scale factor and measurements of corresponding parts (angles, side lengths, perimeters, areas).
   b) Determine if two figures are similar and justify the conclusion by examining corresponding side lengths, angles, perimeters, and area.
   c) *Interpret and solve scaling problems involving various mathematical contexts (e.g. indirect measurement, scale models).

3. Connect the constant rate of change in a proportional relationship to the concept of slope of a line.
   a) *Represent proportional relationships using graphs, tables, verbal descriptions, and equations, and make connections among the representations.
   b) *Determine the slope/rate of change of a line corresponding to the graph of a proportional relationship, recognize that slope is the same between any two points on the line, and that similar triangles may be used to demonstrate constant slope.
   c) *Determine the unit rate in a proportional relationship and relate it to the slope of the associated line.

Grade 7, Core Content C: Surface Area and Volume **20%**

By decomposing two- and three-dimensional shapes into smaller, component shapes, students find surface areas and develop and justify formulas for the surface areas and volumes of prisms and cylinders. As students decompose prisms and cylinders, they develop and understand formulas for their volumes (Volume = Area of base × Height) and surface areas. They apply these formulas in problem solving to determine volumes of prisms and cylinders. Students investigate how surface area and volume are affected when one or more dimensions of a figure are changed. They select appropriate two- and three-dimensional shapes to model real-world situations, create two-dimensional drawings of three-dimensional figures, and solve a variety of problems (including multi-step problems) involving surface areas and volumes of prisms and cylinders.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Describe the components of two- and three-dimensional shapes.
   a) Know the names of the sides of a right triangle (legs and hypotenuse); dimensions of a circle or sphere (radius, diameter); surfaces of rectangular prisms (base, faces, vertices, edges).
   b) Describe the surfaces of common three-dimensional shapes (e.g., base of a cylinder is a circle; face of a cylinder is a rectangle) including cylinders, cones, rectangular prisms, pyramids.
   c) Describe the cross section (parallel to the base) of common three-dimensional figures (e.g., cylinder, cone, rectangular prism).
   d) Name the measurable features of three-dimensional shapes (size of base, height, surface area, volume) and know the types of units used to measure each feature.

2. Develop formulas for measuring surface area and volume of common three-dimensional figures.
   a) *Describe the relationships between the measurements of three-dimensional figures and the measures of related two-dimensional figures (e.g., the volume of a rectangular prism can be found by multiplying the base area by the height.)
   b) *Recognize and draw two-dimensional representations (isometric and perspective drawings) of three-dimensional figures.
   c) Determine the surface area and volume of right prisms and right cylinders.
   d) *Describe the relationship between the volume of right prisms and pyramids, and cylinders and cones.
   e) Describe how surface area and volume are affected when a figure's linear dimensions are changed by a scale factor (whole number or unit fraction).
   f) Solve single- and multi-step word problems involving surface area or volume using appropriate units of measure.

3. Use the term "square root" (or "cube root") to denote the length of sides of squares (or cubes) with given volume.
   a) Find the length of a side of a square given its area and denote the side length as the "square root" of the area. Likewise, find the length of a side of a cube given its volume and denote the side length as the "cube root" of the volume.
   b) Estimate square roots of numbers less than 225 and cube roots of numbers less than 1000 between two whole numbers.
   c) *Recall the square roots of perfect squares between 1 and 100 and the cube roots of perfect cubes from 1 through 1000.
   d) *Solve area and volume problems using square and cube roots.

Grade 7, Core Concept D: Experimental and Theoretical Probability **10%**

Students conduct one-stage experiments, use experimental data to estimate probabilities, and, where possible, compare experimental and theoretical probabilities and examine experimental probability in the long run. Students recognize that repetitions of an experiment may result in different outcomes and describe the variation in outcomes produced in an experiment. Students recognize that small samples are often not representative of the population from which they are drawn, and with the collection of more data the experimental probability of a particular outcome approaches the theoretical probability.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Conduct one-stage experiments to estimate the likelihood of a simple event, compare the experimental probability with an easily identifiable theoretical probability, describe and compare the likelihood of events.
   a) Determine the sample space for a given one-stage experiment using lists, tables, and tree diagrams to represent all possible outcomes.
   b) Within a given context, determine the theoretical probability of an event and its *complement* given a sample space.
   c) *Recognize that with the collection of more data, the experimental probability of a particular outcome approaches the theoretical probability and that although probability cannot determine an individual outcome, it can be used to predict the frequency of an outcome.
   d) Use experimental data to estimate the probability of an event when the theoretical probability is unknown.

GRADE 8

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Grade 8, Core Content A: Real Numbers, Linear Equations, Inequalities and Functions 35%

Students extend their knowledge of rational numbers to negative rational numbers and irrational numbers. Students build on their work with proportions and linear equations to work with a broader set of linear relationships and learn about functions. Students understand that the slope of a line is a constant rate of change. They move flexibly between and among verbal, tabular, graphical, and algebraic representations of functions (recognizing that tabular and graphical representations are usually only partial representations). Students describe how the slope and y-intercept of a function appear in the tabular, graphical, and algebraic representations of the function.

Grade 8, Core Content B: Transformations 15%

Students investigate the effects of transformations on geometric figures. They represent translations, reflections, rotations, and simple compositions of these transformations using sketches and coordinates, and explain the effects of these transformations. Students recognize and identify corresponding parts of the pre-image and image and recognize that these figures are similar. Students understand that symmetry results from transformations and identify lines of symmetry and angles of rotation.

Grade 8, Core Content C: Distance and Angle Measurement 20%

Students use facts about distance and angles to describe and analyze figures and situations in two- and three-dimensional space and to solve problems, including those with multiple steps. They explore the relationships between pairs of angles formed when parallel lines are cut by a transversal and use these relationships to find unknown measures of angles. Students connect geometry and measurement as they use relationships involving the sides and angles of triangles to finding unknown measures. Students explain why the Pythagorean theorem is valid by using a variety of methods—for example, by decomposing a square in two different ways. They apply the Pythagorean theorem to find distances between points in the Cartesian coordinate plane, to measure lengths and to analyze polygons and polyhedra.

Grade 8, Core Content D: Bivariate Data 15%

Students formulate questions involving two attributes, design experiments, collect, organize, and summarize bivariate data using numerical and graphical displays. Students quantify the strength of association between two variables, develop simple models for association, and understand basic interpretations of measures of association.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Grade 8, Core Content A: Real Numbers, Linear Equations, Inequalities and Functions 35%

Students extend their knowledge of rational numbers to negative rational numbers and irrational numbers. Students build on their work with proportions and linear equations to work with a broader set of linear relationships and learn about functions. Students understand that the slope of a line is a constant rate of change. They move flexibly between and among verbal, tabular, graphical, and algebraic representations of functions (recognizing that tabular and graphical representations are usually only partial representations). Students describe how the slope and y-intercept of a function appear in the tabular, graphical, and algebraic representations of the function.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Represent situations and solve problems that involve linear equations, inequalities and functions.
   a) Solve equations with rational coefficients that involve linear relationships.
   b) Solve single- and multi-step word problems involving linear equations and verify the solutions.
   c) Solve inequalities with rational coefficients that involve non-linear relationships.
   d) Represent a linear function with a verbal description, table, graph, or symbolic expression, and make connections among these representations.
   e) Determine the slope and y-intercept of a linear function described by a symbolic expression, table, verbal description or graph.
   f) Interpret the slope and y-intercept of a graph of a linear function representing a contextual situation.
   g) *Identify the domain and range of a function.

2. Develop an understanding of the real number system, including the notation associated with negative numbers, irrational numbers, and large and small numbers.
   a) Recognize that the set of real numbers is made up of rational and irrational numbers and that they can be represented by points on a number line.
   b) Identify a real number as rational or irrational based on the definition of rational numbers (numbers that can be expressed in the form a/b where a and b are integers and b is not equal to 0).
   c) Convert rational numbers to terminating or repeating decimal form and use appropriate notation to indicate the repeating digit sequence.
   d) Extend/adapt algorithms for computing with positive rational numbers to negative rational numbers.
   e) Represent and solve problems involving rational numbers and judge the reasonableness of solutions.
   f) *Read, write, compare and order numbers represented in scientific notation using positive and negative integer exponents for powers of 10, and interpret applications of scientific notation. □

Grade 8, Core Content B: Transformations **15%**

Students investigate the effects of transformations on geometric figures. They represent translations, reflections, rotations, and simple compositions of these transformations using sketches and coordinates, and explain the effects of these transformations. Students recognize and identify corresponding parts of the pre-image and image and recognize that these figures are similar. Students understand that symmetry results from transformations and identify lines of symmetry and angles of rotation.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Apply knowledge of transformations in relating objects and in their effects on figures.
   a) Locate the images of figures produced by transformations including translations, reflections about a vertical or horizontal line, rotations about the origin, and simple composition of these transformations.
   b) *Describe the effects of transformations including translations, reflections about a vertical or horizontal line, rotations about the origin, and simple composition of these transformations.
   c) Determine the coordinates of the image of a figure produced after a translation, a reflection about a vertical or horizontal line, or a rotation of a multiple of 90° about the origin of the coordinate plane
   d) Identify the corresponding parts (segments, angles, vertices) of the pre-image and image of a figure and identify the transformation that has occurred.
   e) Draw lines of symmetry and identify the angle of rotation in designs with rotational symmetry.

Grade 8, Core Content C: Distance and Angle Measurement **20%**

Students use facts about distance and angles to describe and analyze figures and situations in two- and three-dimensional space and to solve problems, including those with multiple steps. They explore the relationships between pairs of angles formed when parallel lines are cut by a transversal and use these relationships to find unknown measures of angles. Students connect geometry and measurement as they use relationships involving the sides and angles of triangles to finding unknown measures. Students explain why the Pythagorean theorem is valid by using a variety of methods—for example, by decomposing a square in two different ways. They apply the Pythagorean theorem to find distances between points in the Cartesian coordinate plane, to measure lengths and to analyze polygons and polyhedra.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Develop and apply the Pythagorean Theorem to solve for the lengths of sides in rights triangles and related measurement problems.
   a) *State the Pythagorean Theorem and justify it using a variety of methods.
   b) Apply the Pythagorean Theorem and its converse to solve problems.
   c) Apply the Pythagorean Theorem to determine the distance between two points on the coordinate plane.

2. Solve problems involving angle measure.
   a) Identify pairs of angles as complementary, supplementary, adjacent, or vertical and find missing angle measures by using these relationships.
   b) Find the measure of angles formed when parallel lines are cut by a transversal and at least one angle measure is given.
   c) Identify, state, and apply the Angle-Sum properties for triangles and other polygons.

Grade 8, Core Content D: Bivariate Data **15%**

Students formulate questions involving two attributes, design experiments, collect, organize, and summarize bivariate data using numerical and graphical displays. Students quantify the strength of association between two variables, develop simple models for association, and understand basic interpretations of measures of association.

Learning Goals/Performance (*Locally Assessed) Indicators:

1. Formulate questions involving two attributes, design experiments, collect, organize, and summarize bivariate data using numerical and graphical displays to represent the data.
   a) *Design experiments and collect bivariate (two-variable) data to answer a question, classifying each attribute as a categorical or numerical variable.
   b) Identify, describe, and construct appropriate displays (*two-way tables*, parallel *box plots* or back-to-back stem-and-leaf plots, and *scatterplots*) for bivariate data.
   c) Summarize data using statistics including five-number summaries and *Inter-Quartile Range (IQR)*.

2. Quantify the strength of association between two variables using a variety of tools, develop simple models for association between two numerical variables, and understand basic interpretations of measures of association.
   a) Quantify the strength of association between two variables and develop simple models using contingency tables (for categorical data) and the Quadrant Count Ratio (for numerical data) and simple models for examining the association between two numerical values such as simple lines (e.g., median-median line).
   b) *Describe the relationship between the two variables, the effects of outliers on the observed relationship, and distinguish between an “association” and “cause and effect.”

Core Content, Learning Goals and Performance Indicators

High School

ALGEBRA 1

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Algebra 1, Core Content A: Linear and Nonlinear Relationships **15%**

Students identify relationships with a constant rate of change that can be represented by linear functions. They model these situations using algebraic, graphical, tabular, and verbal representations (recognizing that tabular and graphical representations are usually only partial representations of the function). They move flexibly between and among representations and solve contextual problems. They compare linear and nonlinear relationships using rates of change and graphical representations. They apply the terminology and symbols associated with expressions, functions, and linear equations, including domain, range, slope, intercepts, independent and dependent variables.

Algebra 1, Core Content B: Linear Expressions, Equations, Inequalities, and Functions **30%**

Students distinguish among different uses of unknowns/variables. They find equivalent expressions and equations. They construct, represent, solve, and interpret solutions of linear equations, absolute value equations, linear inequalities, and systems of linear equations, including those derived from contextual situations.

Algebra 1, Core Content C: Nonlinear Functions **30%**

Students identify and classify non-linear relationships. They represent simple contextual phenomena using exponential and quadratic functions and solve equations involving these functions with a variety of techniques. They determine the reasonableness of solution(s) and extend their understanding of the real number system through solving problems in algebraic situations.

Algebra 1, Core Content D: Trend Lines and Correlation **10%**

Students develop models for trends in bivariate data using both median-fit lines and least-squares regression lines. They use the correlation coefficient to measure linear association in scatterplots. Students examine the effects of outliers on the models for trend and on correlation. Students extend their understanding of data analysis through modeling and interpreting problems related to contextual situations.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Algebra 1, Core Content A: Linear and Nonlinear Relationships **15%**

Students identify relationships with a constant rate of change that can be represented by linear functions. They model these situations using algebraic, graphical, tabular, and verbal representations (recognizing that tabular and graphical representations are usually only partial representations of the function). They move flexibly between and among representations and solve contextual problems. They compare linear and nonlinear relationships using rates of change and graphical representations. They write and interpret equations for a They apply the terminology and symbols associated with expressions, functions, and linear equations, including domain, range, slope, intercepts, independent and dependent variables.

Learning Goals and Performance (Locally Assessed) Indicators:

1) Identify functions as linear or nonlinear based on their graphical behavior and rates of change, describe functions using appropriate notation and terminology, and evaluate functions at given points.
   a) Determine and justify whether a relationship is a function by using a graph or a verbal description of the relationship.
   b) Compare linear and nonlinear relationships by examining rates of change from verbal descriptions, tables of values, graphical representations, and symbolic forms.
   c) Simplify absolute value expressions and solve simple absolute value equations.
   d) Describe characteristics of piecewise-linear functions, including absolute value (linear) functions, give examples of situations in which they arise and graph these functions over the appropriate domain.
   e) Evaluate a function at a specific point, e.g. given the function $f(x) = 3x - 2$, find $f(x)$.

2) Use linear functions to represent, interpret and solve problems. Understand the relationship between slopes of lines in the plane that are parallel, perpendicular, or neither.
   a) Represent linear patterns or arithmetic sequences using verbal rules and symbolic expressions such as kx (proportional relationships) and $ax + b$ (more-general linear relationships).
   b) Analyze a contextual situation, determine whether a linear function can describe the situation, and if so, write the linear function.
   c) Write equations and graph lines given particular information (e.g., two points on a line or slope and one point on the line).
   d) Identify the slope and x- and y-intercepts of a line given the equation of the line.
   e) Know the relationship between the slopes of parallel lines and between perpendicular lines and use these relationships to identify and/or write equations for lines parallel or perpendicular to a given line.

Algebra 1, Core Content B: Linear Expressions, Equations, Inequalities, and Functions 30%

Students distinguish among different uses of unknowns/variables. They find equivalent expressions and equations. They construct, represent, solve, and interpret solutions of linear equations, absolute value equations, linear inequalities, and systems of linear equations, including those derived from contextual situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Represent linear patterns using expressions, equations, inequalities, and function notation, as appropriate. Generate equivalent forms of these representations to interpret and solve problems.
   a) Describe the meaning of expressions and functions of the form $ax + b$ and f(x)=ax + b, and interpret the changes resulting from different values of the parameters a and b.
   b) Justify that two algebraic expressions, equations, or inequalities are equivalent using the properties of equality and inequality, as well as the commutative, associative, inverse, identity, and distributive properties.
   c) Use equivalent forms of linear expressions, equations and functions including slope-intercept, point-slope, intercept, and general (standard) forms, recognizing that different representations of a linear relationship reveal different information about a given situation.

2. Distinguish among the different uses of unknowns/variables, parameters, constants, and equations.
   a) Identify and distinguish among parameters and the independent and dependent unknowns.
   b) Distinguish among and be able to solve different types of equations that can be constructed by equating linear expressions, including identities (e.g., $x + x = 2x$); equations for which there is no solution (e.g., $x + 1 = x + 2$); formulas (e.g., $C = \pi d$); equations where the solution is unique (e.g., $2x + 3 = 16$); and equations relating two unknowns (e.g., $y = 3x + 7$).
   c) Solve for terms in an arithmetic (linear) sequence using given verbal rules or symbolic expressions (explicit and recursive).

3. Construct, solve, and interpret solutions of linear equations, linear inequalities, and systems of linear equations (limited to two equations with two unknowns) in contextual situations.
   a) Construct a linear equation or linear inequality to model a contextual situation.
   b) Solve linear equations, absolute value (linear) equations and inequalities, and linear inequalities using symbolic methods, graphs, tables and technology.
   c) *Construct a system of linear equations modeling a contextual situation.
   d) *Analyze, solve, and explain the reasoning used to solve a system of linear equations and describe the nature of the solutions (no solution, one solution, or infinitely many solutions).

Algebra 1, Core Content C: Nonlinear Functions **30%**

Students identify and classify non-linear relationships. They represent simple contextual phenomena using exponential and quadratic functions and solve equations involving these functions with a variety of techniques. They determine the reasonableness of solution(s) and extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Identify certain nonlinear relationships and classify them as exponential relationships, quadratic relationships, or relationships of the form $y = \frac{k}{x}$, based on rates of change in tables, symbolic forms, or graphical representations. Recognize that multiplying linear factors produces nonlinear relationships.
   a) Identify nonlinear (exponential, quadratic, and equations of the form $y = \frac{k}{x}$) relationships in graphical or tabular displays through an examination of successive differences, ratios, symbolic forms, or graphical properties.
   b) *Solve for terms in a geometric (exponential) sequence using given verbal rules or symbolic expressions (explicit and recursive).
   c) Multiply a pair of linear expressions and interpret the result of the operation.
   d) Recognize exponential functions from their verbal descriptions and tabular, graphical or symbolic representations, and move flexibly between and among these representations.
   e) Describe the effects of changes in the coefficient, base, and exponent on the growth described by an exponential function.

2. Represent and interpret simple exponential and quadratic functions based on contextual phenomena using tables, symbolic forms, or graphical representations and solve equations related to these functions.
   a) Find integer powers of rational numbers, explain the meaning of integer powers of unknowns in expressions, and apply the basic laws of exponents (e.g., $a^m \bullet a^n = a^{m+n}$).
   b) Apply laws of exponents in problem situations (e.g. scientific notation).
   c) Distinguish among general forms for exponential equations ($y = b^x$, $y = a \bullet b^x$) and quadratic equations ($y = x^2$, $y = -x^2$, $y = ax^2$, $y = x^2 + c$, $y = ax^2 + c$), and describe how the values of *a, b,* and *c* affect their graphical representations.
   d) Describe the effects of changes in the coefficient, base, and exponent on the growth described by an exponential function.
   e) Represent simple exponential and quadratic equations using manipulative models, verbal descriptions, tables, graphs and technology.
   f) *Identify points of intersection of the graph of a quadratic equation of the form $y = ax^2$ or exponential equation of the form $y = ab^x$ and the graph of a line of the form $y = k$ with and without technology. Relate the points of intersection to the solutions of the quadratic equation $ax^2 = k$ or exponential equation $y = ab^x$.
   g) Factor simple quadratic expressions (limited to greatest common factor, perfect-square trinomials, difference of squares, and quadratics of the form $x^2 + bx + c$ that factor over the integers), and apply the zero-product property to solve quadratic equations.

Algebra 1, Core Content D: Trend Lines and Correlation **10%**

Students develop models for trends in bivariate data using both median-fit lines and least-squares regression lines. They use the correlation coefficient to measure linear association in scatterplots. Students examine the effects of outliers on the models for trend and on correlation. Students extend their understanding of data analysis through modeling and interpreting problems related to contextual situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Model association of bivariate numerical data using tables and scatterplots and apply the correlation coefficient to measure and describe linear association. Apply both median-fit lines and least-squares regression lines to describe trends in bivariate data, when appropriate.
   a) Compute the median-median line, by hand, to model a relationship shown in a scatterplot, and interpret the slope and intercept in terms of the original context.
   b) *Generate the least-squares regression line, using technology, to model a relationship shown in a scatterplot.
   c) Interpret the slope and intercept of regression lines in terms of the original context.
   d) *Determine the correlation, using technology, between two numerical unknowns, interpret the correlation, and describe the strengths and weaknesses of the correlation coefficient as a measure of linear association.
   e) Interpolate using trends observed in scatterplots or fitted regression lines, and judge when extrapolating observed trends may be appropriate.
   f) Given a set of data or scatter plot, choose the line of best fit and use the line to make predictions.
   g) Recognize how linear transformations of one-variable data affect mean, median, mode and range.

2. Describe the influence of outliers on correlation and on models for trend.
   a) Identify unusual observations in scatterplots, and conjecture about the effect of such outliers on the strength of the association between the unknowns defining the scatterplot.
   b) *Use technology to investigate and describe the influence outliers may have on a correlation coefficient, on the slope and intercept of a least-squares regression line, and on a median-fit line.

GEOMETRY

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Geometry, Core Content A: Geometric Reasoning, Proof, and Representation **25%**

Students represent geometric figures and investigate a variety of relationships among them, form conjectures, and attempt to verify or reject the conjectures. They develop and apply various methods of proving or disproving conjectures within the axiomatic structure of Euclidean geometry.

Geometry, Core Content B: Similarity and Congruency **25%**

Students identify the properties and perform the mappings of rigid transformations and origin-centered dilations of figures in the plane. They discuss the connections between rigid transformations and origin-centered dilations and the relations of congruence and similarity, respectively. They develop and apply sufficient conditions for proving planar figures congruent or similar. They apply similarity to solve a variety of contextual problems.

Geometry, Core Content C: Direct and Indirect Measurement **25%**

Students justify and apply the measurement formulas associated with one-, two-, and three-dimensional geometric objects. They prove and apply the Pythagorean Theorem, right-triangle trigonometric ratios, and proportionality relationships in structuring and solving indirect measurement problems. Students extend their understanding of the real number system through solving problems in geometric situations.

Geometry, Core Content D: Conditional Probability and Independence **10%**

Students determine the sample space for multi-stage experiments. They distinguish between independent and dependent events and compute their probabilities. Students design and use simulations to solve contextual probability problems.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Geometry, Core Content A: Geometric Reasoning, Proof, and Representation **25%**

Students represent geometric figures and investigate a variety of relationships among them, form conjectures, and attempt to verify or reject the conjectures. They develop and apply various methods of proving or disproving conjectures within the axiomatic structure of Euclidean geometry.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Describe and apply inductive and deductive reasoning to form conjectures and attempt to verify or reject them through developing short sequences of geometric theorems within a local axiomatic system or by developing counterexamples.
   a) Describe the structure of and relationships within an axiomatic system (undefined terms, defined terms, axioms/postulates, methods of reasoning, and theorems).
   b) Form conjectures based on exploring geometric situations with and without technology.
   c) Prove, directly or indirectly, that a valid mathematical statement is true. Develop a counterexample to refute an invalid statement.
   d) Identify flaws or gaps in the reasoning supporting an argument.
   e) Formulate and investigate the validity of the converse, inverse, or contrapositive of a conditional statement.

2. Apply mathematical methods of proof to develop justifications for basic theorems of Euclidean geometry. Develop the triangle angle-sum and angle-measure theorems for polygons, and the triangle- and angle-inequality theorems.
   a) Justify statements about angles formed by perpendicular lines and transversals of parallel lines.
   b) Justify and apply properties of circles (e.g., perpendicularity of tangent and radius, angle inscribed in a circle).
   c) Organize and present direct and indirect proofs using two-column, paragraph, and flow-chart formats.

3. Use a variety of representations to describe geometric objects and to analyze relationships among them.
   a) Use coordinates and algebraic representations (e.g., distances, points that divide segments in specified ratios, slope) to describe and define figures.
   b) Use nets, drawings (including isometric), vertex-edge graphs, models, and technologically created images to represent geometric objects from different perspectives and analyze relationships among them.

Geometry, Core Content B: Similarity and Congruency **25%**

Students identify the properties and perform the mappings of rigid transformations and origin-centered dilations of figures in the plane. They discuss the connections between rigid transformations and origin-centered dilations and the relations of congruence and similarity, respectively. They develop and apply sufficient conditions for proving planar figures congruent or similar. They apply similarity to solve a variety of contextual problems.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Identify and apply transformations of figures in the coordinate plane and discuss the results of these transformations. Identify and justify congruence by establishing sufficient conditions and by finding a congruence-preserving rigid transformation between the figures. Solve problems involving congruence in a variety of contexts.
   a) Identify and differentiate among sufficient conditions for congruence of triangles (SSS, SAS, ASA, AAS, and HL) and apply them to identify congruent triangles.
   b) Represent translations, line reflections, rotations, and origin-centered dilations of objects in the coordinate plane by using sketches, coordinates, and function notation, and explain the effects of these transformations.
   c) Recognize and identify corresponding parts of congruent and similar figures after transformation(s).
   d) Use coordinate geometry and rigid transformations (reflections, translations, and rotations) to establish congruence of figures.
   e) Apply geometric properties and relationships in solving multi-step problems in two and three dimensions.

2. Identify similar figures and justify similarity by establishing sufficient conditions and by finding an origin-centered dilation between the figures. Solve problems involving similarity in a variety of contexts.
   a) Identify conditions for establishing similarity of triangles (SAS, SSS, AA), and apply them, noting that congruence is a special case of similarity.
   b) Create a representation of a figure similar to a specified figure given their similarity ratio and use origin-centered dilations to describe and investigate similarities.

Geometry, Core Content C: Direct and Indirect Measurement **25%**

Students justify and apply the measurement formulas associated with one-, two-, and three-dimensional geometric objects. They prove and apply the Pythagorean Theorem, right-triangle trigonometric ratios, and proportionality relationships in structuring and solving indirect measurement problems. Students extend their understanding of the real number system through solving problems in geometric situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Develop two- and three-dimensional measurement formulas for perimeter/circumference, area, and volume and apply these formulas and other geometric properties to solve problems involving simple and composite geometric objects.
   a) Derive and use area formulas for quadrilaterals and regular polygons.
   b) Link the surface area of prisms and cylinders to the sum of the areas of their bases and lateral surfaces using planar nets to illustrate and sum the relevant measures.
   c) Develop the triangle angle-sum and angle-measure theorems for polygons, and the triangle- and angle-inequality theorems
   d) Identify and find the measures of angles formed by segments in three-dimensional figures, extending right triangle and isosceles/equilateral triangle relationships to study the planar faces of three-dimensional objects.
   e) Apply and link formulas to solve problems involving area, perimeter, volume, and surface area of prisms, cylinders, pyramids, cones, spheres, and composite figures.
   f) Apply the ratio of similitude to determine perimeter, area, and volume measurements of similar figures.
   g) Determine arc lengths of circles and areas of sectors of circles using proportions.
   h) Use geometric models to represent and solve mathematical or real-world problems.

2. Prove and apply the Pythagorean Theorem and its converse. Develop and apply the distance formula, properties of special right triangles, properties of proportions, and basic trigonometric ratios.
   a) Apply the distance formula to determine the distance between points in the coordinate plane.
   b) Apply properties of 30°-60°-90° and 45°-45°-90° triangles and the Pythagorean Theorem to determine triangle side lengths.
   c) Prove and apply the Pythagorean Theorem and its converse.
   d) Use properties of proportionality and similarity in solving contextual problems in two- and three-dimensional settings.
   e) *Apply the sine, cosine, and tangent trigonometric ratios to determine lengths and angle measures in right triangles.

Geometry, Core Content D: Conditional Probability and Independence **10%**

Students determine the sample space for multi-stage experiments. They distinguish between independent and dependent events and compute their probabilities. Students design and use simulations to solve contextual probability problems.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Determine the sample space for multi-stage experiments (e.g., tossing three coins) using a variety of representations and employ systematic counting approaches to determine the number of possible outcomes. Distinguish between independent and dependent compound events, and compute their probabilities using a variety of representations and applying the multiplication rule for probability.
   a) Use set notation to convey information and use Venn diagrams to illustrate given information about sets.
   b) Determine the number of possible outcomes in the sample space for multi-stage experiments using a variety of representations (e.g., tree diagrams, lists) and systematic counting approaches including permutations, combinations, and the multiplication rule for counting (Fundamental Property of Counting).
   c) Distinguish between independent and dependent compound events, and explain the idea of conditional probability using two-way tables and appropriate notation and terminology (e.g. union)
   d) Use tables, trees, and geometric representations (area models) to describe and apply the multiplication rule for probability to compute probabilities for independent and for dependent compound events.

2. Develop, use, and interpret simulations to estimate probabilities for events where theoretical values are difficult or impossible to compute. Recognize that simulation results are likely to differ from one run of the simulation to the next, and that results of the simulation tend to converge as the number of trials increases.
   a) *Describe and conduct a simulation by identifying the components and assumptions in a problem, selecting a device to generate chance outcomes, defining a trial, specifying the number of trials and carrying out the simulation.
   b) Summarize data from a simulation using appropriate graphical and numerical summaries, develop an estimate for the probability of an event where theoretical values are difficult or impossible to compute, and discuss the effect of the number of trials on the estimated probability of the event.

ALGEBRA 2

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Algebra 2, Core Content A: Polynomial Expressions, Equations, and Functions **30%**

Students extend understanding of functions from linear settings to include polynomial functions, operations on these functions, and the solution of polynomial equations using complex numbers. They use polynomials (with a focus on quadratics) to model situations with graphical and symbolic representations. They flexibly move among polynomial representations (e.g. factored form, standard form) to represent and discuss the qualitative behavior of the associated functions. Students extend their understanding of the real number system through solving problems in algebraic situations.

Algebra 2, Core Content B: Exponential, Logarithmic, and Other Functions **30%**

Students develop exponential, logarithmic, and other non-linear functions (rational, radical, absolute value, and piecewise-defined) to represent, investigate, and solve mathematical and contextual problems. They extend their understanding of the real number system through solving problems in algebraic situations.

Algebra 2, Core Content C: Systems of Equations and Inequalities and Matrices **15%**

Students construct, solve, and interpret solutions of systems of equations and inequalities in two or more unknowns. They represent cross-categorized data in matrices and perform operations on matrices to model and interpret problem situations. They model problem situations with systems of equations and solve with technology. Students extend their understanding of the real number system through solving problems in algebraic situations.

Algebra 2, Core Content D: Binomial Theorem and Probability **10%**

Students use the binomial expansion theorem and recognize the connection to Pascal's triangle, probability, and combinatorics. Students recognize a binomial probability setting, develop and graph the binomial probability distribution, and compute the probability distribution for a binomial count. They extend their understanding of the real number system through problem solving situations.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Algebra 2, Core Content A: Polynomial Expressions, Equations, and Functions **30%**

Students extend understanding of functions from linear settings to include polynomial functions, operations on these functions, and the solution of polynomial equations using complex numbers. They use polynomials (with a focus on quadratics) to model situations with graphical and symbolic representations. They flexibly move among polynomial representations (e.g. factored form, standard form) to represent and discuss the qualitative behavior of the associated functions.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Operate with monomials, binomials, and polynomials. Apply the composition of functions to model and solve problems.
   a) Add, subtract, multiply, and divide polynomial expressions.
   b) Analyze and graph polynomial functions by identifying the intercepts, zeros, domain and range, turning points, and end behavior.
   c) Use factoring (including differences of squares, sum and difference of cubes and other factorable polynomials) to transform expressions and solve problems.
   d) Apply the Factor Theorem to solve polynomial equations and the Remainder Theorem to evaluate polynomial functions.
   e) Apply the Fundamental Theorem of Algebra to determine the number of real and complex roots of a polynomial function.
   f) Apply the composition of functions to model and solve problems, and explain the results.

2. Represent, compare, and move flexibly between and among representations (including graphic, symbolic, and tabular) to interpret, solve problems, and verify solutions involving quadratic equations. Develop and apply the quadratic formula.
   a) Write, interpret, apply and move flexibly between and among equivalent forms of quadratic expressions, equations and functions, including the general (standard) and vertex forms, recognizing that equivalent forms for a quadratic relationship reveal different information about a given situation.
   b) Determine reasonable domain and range values for quadratic functions within a context, and test the reasonableness of solutions to quadratic equations (zeros of quadratic functions).
   c) Graph a quadratic function, and recognize the relationships between the coefficients of a quadratic function and characteristics of its graph (e.g., shape, position, intercepts, zeros, maximum, minimum, symmetry, vertex).
   d) Solve quadratic equations and inequalities by factoring, completing the square, and with technology. Interpret and justify solutions in terms of the original problem context and represent solutions graphically.
   e) Apply the quadratic formula to solve quadratic equations and interpret the nature of the roots.

3. Represent and apply properties of complex numbers.
   a) Define, plot, and compute with complex numbers.
   b) *Describe how the associative, commutative, and distributive properties of operations on real numbers extend to operations on complex numbers.
   c) Solve quadratic equations with real coefficients over the set of complex numbers.

Algebra 2, Core Content B: Exponential, Logarithmic, and Other Functions **30%**

Students develop exponential, logarithmic, and other non-linear functions (rational, radical, absolute value, and piecewise-defined) to represent, investigate, and solve mathematical and contextual problems. They extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Represent geometric or exponential growth with exponential functions and equations, and apply such functions and equations to solve problems in contextual situations.
   a) Simplify expressions with real (both rational and irrational) exponents and relate the expressions with rational exponents to equivalent radical expressions. Describe the effects of changes in the coefficient, base, and exponent on the growth described by an exponential function.
   b) Analyze a problem situation modeled by an exponential function (e.g. exponential growth and decay, compound interest), formulate an equation or inequality, and solve the problem.
   c) Graph and analyze the behavior of exponential functions.

2. Define logarithmic functions and use them to solve problems in contextual situations.
   a) Define a logarithm as a solution to an exponential equation, and recognize the inverse relationship between logarithmic functions and exponential functions, showing this relationship graphically.
   b) Solve problems by applying properties of logarithms (for example, $\log_b xy = \log_b x + \log_b y$) to construct equivalent forms of a logarithmic expression.
   c) Apply the inverse relationship between exponential and logarithmic functions to solve problems in context.

3. Interpret and represent rational and radical functions and solve rational and radical equations.
   a) Model and solve problems using direct, inverse, joint, and combined variation.
   b) Add, subtract, multiply, divide, and simplify rational expressions with linear and quadratic denominators. Evaluate rational functions.
   c) Model problem situations by constructing equations and inequalities based on rational and radical functions, solve using a variety of methods, and interpret solutions in terms of the problem situation.
   d) Describe the graphs of rational and radical functions, including limitations on the domains and ranges, and asymptotic behavior.
   e) Use properties of radicals to simplify radicals and radical expressions, to solve radical equations and to identify extraneous roots when they occur.

4. Interpret and model step and other piecewise-defined (i.e. linear and quadratic) functions, including absolute value functions.
   a) Analyze a problem situation to determine or interpret reasonable domain and range values for piecewise-defined functions representing the situation.
   b) Interpret, construct and apply step functions (e.g., greatest integer/floor) and other piecewise-defined functions, including absolute value functions, to model and solve problems.
   c) Move flexibly between and among verbal, graphical, tabular, and symbolic representations of step functions and other piecewise-defined functions, including absolute value functions.

Algebra 2, Core Content C: Systems of Equations and Inequalities and Matrices **15%**

Students construct, solve, and interpret solutions of systems of equations and inequalities in two or more unknowns. They represent cross-categorized data in matrices and perform operations on matrices to model and interpret problem situations. They model problem situations with systems of equations and solve with technology. Students extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Construct, solve, and interpret solutions of systems of equations and inequalities (including all combinations of linear and quadratic equations) in two unknowns.
   a) Construct a system of equations or inequalities in two unknowns to represent a contextual setting.
   b) Analyze and explain the reasoning used to solve systems of linear equations and inequalities in two unknowns.
   c) Solve a system of equations or inequalities in two unknowns using a variety of methods, and interpret the meaning of the solution.

2. Represent and interpret cross-categorized data in matrices, develop properties of matrix addition, and use matrix addition and its properties to solve problems.
   a) *Represent numerical or relational data categorized by two or more unknowns in a matrix and label the rows and columns. Interpret the meaning of a particular entry in a matrix in terms of the labels of its row and column.
   b) *Use matrix row and column sums to analyze data.

3. Multiply matrices, verify the properties of matrix multiplication, and use the matrix form for a system of linear equations to structure and solve systems consisting of two or three linear equations in two or three unknowns, respectively, with technology.
   a) *Verify the properties of matrix multiplication and multiply matrices to solve problems.
   b) Construct a system of linear equations modeling a contextual situation, and represent the system as a matrix equation ($Ax = b$), that is:

$$\begin{aligned} ax + by &= c \\ dx + ey &= f \end{aligned} \quad \Leftrightarrow \quad \begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$$

   c) *Solve a system consisting of two or more linear equations in two or more unknowns, respectively, by solving the related matrix equation $Ax = b$, using technology to find $x = A^{-1}b$.

Algebra 2, Core Content D: Binomial Theorem and Probability **10%**

Students use the binomial expansion theorem and recognize the connection to Pascal's triangle, probability, and combinatorics. Students recognize a binomial probability setting, develop and graph the binomial probability distribution, and compute the probability distribution for a binomial count. They extend their understanding of the real number system through problem solving situations.

Learning Goals and Performance (*Locally Assessed) Expectations:

1. Develop facility with the binomial expansion theorem and its connections to Pascal's triangle, probability, and combinatorics. Recognize a binomial probability setting and compute the probability distribution for a binomial count.
   a) *Use the binomial expansion theorem, and relate the expansion of $(a + b)^n$ with the possible outcomes of a binomial experiment and the n^{th} row of Pascal's triangle.
   b) *Recognize a binomial probability setting, and develop and graph the probability distribution for a binomial count.

2. Identify settings in which the normal distribution may be useful, and describe characteristics of the normal distribution. Use graphical displays and the empirical rule to solve problems.
   a) *Identify settings in which the normal distribution may be useful, and describe characteristics of a normal distribution including mean, standard deviation, and z-scores.
   b) *Use graphical displays and the empirical rule to evaluate the appropriateness of the normal model for a given set of data, and use the empirical rule to estimate the probability that an event will occur in a specific interval that can be described in terms of whole numbers of standard deviations about the mean.

INTEGRATED MATHEMATICS 1

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Integrated Mathematics 1, Core Content A: Linear and Nonlinear Relationships **15%**

Students identify relationships with a constant rate of change that can be represented by linear functions. They model these situations using algebraic, graphical, tabular, and verbal representations (recognizing that tabular and graphical representations are usually only partial representations of the function). They move flexibly between and among representations and solve contextual problems. They compare linear and nonlinear relationships using rates of change and graphical representations. They write and interpret equations for a They apply the terminology and symbols associated with expressions, functions, and linear equations, including domain, range, slope, intercepts, independent and dependent variables.

Integrated Mathematics 1, Core Content B: Linear Expressions, Equations, Inequalities, and Functions **25%**

Students distinguish among different uses of unknowns/variables. They find equivalent expressions and equations. They construct, represent, solve, and interpret solutions of linear equations, absolute value equations, linear inequalities, and systems of linear equations, including those derived from contextual situations. They determine the reasonableness of the solution(s) and extend their understanding of the real number system through solving problems in algebraic situations.

Integrated Mathematics 1, Core Content C: Nonlinear Functions **20%**

Students identify and classify non-linear relationships. They represent simple contextual phenomena using exponential and quadratic functions and solve equations involving these functions with a variety of techniques. They determine the reasonableness of the solution(s) and extend their understanding of the real number system through solving problems in algebraic situations.

Integrated Mathematics 1, Core Content D: Geometric Reasoning and Relationships **15%**

Students represent geometric objects and investigate a variety of relationships among them, form conjectures and attempt to verify or reject the conjectures. They develop and apply various methods of proving statements or disproving conjectures within the axiomatic structure of Euclidean geometry.

Integrated Mathematics 1, Core Content E: Trend Lines and Correlation **10%**

Students develop models for trends in bivariate data using both median-fit lines and least-squares regression lines. They use the correlation coefficient to measure linear association in scatterplots. Students examine the effects of outliers on the models for trend and on correlation. Students extend their understanding of data analysis through modeling and interpreting problems related to contextual situations.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Integrated Mathematics 1, Core Content A: Linear and Nonlinear Relationships **15%**

Students identify relationships with a constant rate of change that can be represented by linear functions. They model these situations using algebraic, graphical, tabular, and verbal representations (recognizing that tabular and graphical representations are usually only partial representations of the function). They move flexibly between and among representations and solve contextual problems. They compare linear and nonlinear relationships using rates of change and graphical representations. They write and interpret equations for a They apply the terminology and symbols associated with expressions, functions, and linear equations, including domain, range, slope, intercepts, independent and dependent variables.

Learning Goals and Performance (*Locally Assessed) Indicators:

1) Identify functions as linear or nonlinear based on their graphical behavior and rates of change, describe functions using appropriate notation and terminology, and evaluate functions at given points.
   a) Determine and justify whether a relationship is a function by using a graph or a verbal description of the relationship.
   b) Compare linear and nonlinear relationships by examining rates of change from verbal descriptions, tables of values, graphical representations, and symbolic forms.
   c) Simplify absolute value expressions and solve simple absolute value equations.
   d) Describe characteristics of piecewise-linear functions, including absolute value (linear) functions, give examples of situations in which they arise and graph these functions over the appropriate domain.
   e) Evaluate a function at a specific point, e.g. given the function $f(x) = 3x - 2$, find $f(x)$.

2) Use linear functions to represent, interpret and solve problems. Understand the relationship between slopes of lines in the plane that are parallel, perpendicular, or neither.
   a) Represent linear patterns or arithmetic sequences using verbal rules and symbolic expressions such as kx (proportional relationships) and $ax + b$ (more-general linear relationships).
   b) Analyze a contextual situation, determine whether a linear function can describe the situation, and if so, write the linear function.
   c) Write equations and graph lines given particular information (e.g., two points on a line or slope and one point on the line).
   d) Identify the slope and x- and y-intercepts of a line given the equation of the line.
   e) Know the relationship between the slopes of parallel lines and between perpendicular lines and use these relationships to identify and/or write equations for lines parallel or perpendicular to a given line.

Integrated Mathematics 1, Core Content B: Linear Expressions, Equations, Inequalities, and Functions **25%**

Students distinguish among different uses of unknowns/variables. They find equivalent expressions and equations. They construct, represent, solve, and interpret solutions of linear equations, absolute value equations, linear inequalities, and systems of linear equations, including those derived from contextual situations. They determine the reasonableness of the solution(s) and extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Represent linear patterns using expressions, equations, inequalities, and function notation, as appropriate. Generate equivalent forms of these representations to interpret and solve problems.
   a) Describe the meaning of expressions and functions of the form $ax + b$ and f(x)=ax + b, and interpret the changes resulting from different values of the parameters a and b.
   b) Justify that two algebraic expressions, equations, or inequalities are equivalent using the properties of equality and inequality, as well as the commutative, associative, inverse, identity, and distributive properties.
   c) Use equivalent forms of linear expressions, equations and functions including slope-intercept, point-slope, intercept, and general (standard) forms, recognizing that different representations of a linear relationship reveal different information about a given situation.

2. Distinguish among the different uses of unknowns/variables, parameters, constants, and equations.
   a) Identify and distinguish among parameters and the independent and dependent unknowns.
   b) Distinguish among and be able to solve different types of equations that can be constructed by equating linear expressions, including identities (e.g., $x + x = 2x$); equations for which there is no solution (e.g., $x + 1 = x + 2$); formulas (e.g., $C = \pi d$); equations where the solution is unique (e.g., $2x + 3 = 16$); and equations relating two unknowns (e.g., $y = 3x + 7$).
   c) Solve for terms in an arithmetic (linear) sequence using given verbal rules or symbolic expressions (explicit and recursive).

3. Construct, solve, and interpret solutions of linear equations, linear inequalities, and systems of linear equations (limited to two equations with two unknowns) in contextual situations.
   a) Construct a linear equation or linear inequality to model a contextual situation.
   b) Solve linear equations, absolute value (linear) equations and inequalities, and linear inequalities using symbolic methods, graphs, tables and technology.

Integrated Mathematics 1, Core Content C: Nonlinear Functions **20%**

Students identify and classify non-linear relationships. They represent simple contextual phenomena using exponential and quadratic functions and solve equations involving these functions with a variety of techniques. They determine the reasonableness of the solution(s) and extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Identify nonlinear relationships and classify them as exponential, quadratic, or of the form $y = \frac{k}{x}$, based on rates of change in tables, symbolic forms, or graphical representations. Recognize that multiplying linear factors produces nonlinear relationships.
   a) Identify nonlinear (exponential, quadratic, and equations of the form $y = \frac{k}{x}$) relationships in graphical or tabular displays through an examination of successive differences, ratios, symbolic forms, or graphical properties.
   b) *Solve for terms in a geometric (exponential) sequence using given verbal rules or symbolic expressions (explicit and recursive).
   c) Multiply a pair of linear expressions, and interpret the result of the operation.
   d) Recognize exponential functions from their verbal descriptions and tabular, graphical or symbolic representations, and move flexibly between and among these representations.
   e) Describe the effects of changes in the coefficient, base, and exponent on the growth described by an exponential function.

2. Represent and interpret simple exponential and quadratic functions based on contextual phenomena using tables, symbolic forms, or graphical representations and solves equations related to these functions.
   a) Find integer powers of rational numbers, explain the meaning of integer powers of unknowns in expressions, and apply basic laws of exponents (e.g., $a^m \cdot a^n = a^{m+n}$).
   b) Apply laws of exponents in problem situations (e.g. scientific notation).
   c) Distinguish among general forms for exponential equations ($y = b^x$, $y = a \cdot b^x$) and quadratic equations ($y = x^2$, $y = -x^2$, $y = ax^2$, $y = x^2 + c$, $y = ax^2 + c$), and describe how the values of *a, b,* and *c* affect their graphical representations.
   d) Describe the effects of changes in the coefficient, base, and exponent on the growth described by an exponential function.
   e) Represent simple exponential and quadratic equations using manipulative models, verbal descriptions, tables, graphs and technology.
   f) Factor simple quadratic expressions (limited to the greatest common factor, perfect-square trinomials, difference of squares, and quadratics of the form $x^2 + bx + c$ that factor over the integers), and apply the zero-product property to solve quadratic equations.

Integrated Mathematics 1, Core Content D: Geometric Reasoning and Relationships 15%

Students represent geometric objects and investigate a variety of relationships among them, form conjectures and attempt to verify or reject the conjectures. They develop and apply various methods of proving statements or disproving conjectures within the axiomatic structure of Euclidean geometry.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Use a variety of representations to describe geometric objects and to analyze relationships among them.
   a) *Use coordinates and algebraic representations (e.g., distances, points that divide segments in specified ratios, slope) to describe and define figures.
   b) *Use nets, drawings (including isometric), vertex-edge graphs, models, and technologically created images to represent geometric objects from different perspectives and analyze relationships among them.

2. Develop, test, and provide justifications, based on inductive and deductive methods, for conjectures involving relations of lines, angles, and figures.
   a) *Describe the structure of and relationships within an axiomatic system (undefined terms, defined terms, axioms/postulates, methods of reasoning, and theorems).
   b) *Recognize flaws or gaps in the reasoning supporting an argument.
   c) *Test conjectures about angles, lines, bisectors, polygons (especially triangles and quadrilaterals), circles, and three-dimensional figures.
   d) *Justify statements about angles formed by perpendicular lines and transversals of parallel lines.

Integrated Mathematics 1, Core Content E: Trend Lines and Correlation **10%**

Students develop models for trends in bivariate data using both median-fit lines and least-squares regression lines. They use the correlation coefficient to measure linear association in scatterplots. Students examine the effects of outliers on the models for trend and on correlation. Students extend their understanding of data analysis through modeling and interpreting problems related to contextual situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Model association of bivariate numerical data using tables and scatterplots and apply the correlation coefficient to measure and describe linear association. Apply both median-fit lines and least-squares regression lines to describe trends in bivariate data, when appropriate.
   a) Compute the median-median line, by hand, to model a relationship shown in a scatterplot, and interpret the slope and intercept in terms of the original context.
   b) *Generate the least-squares regression line, using technology, to model a relationship shown in a scatterplot.
   c) Interpret the slope and intercept of regression lines in terms of the original context.
   d) *Determine the correlation, using technology, between two numerical unknowns, interpret the correlation, and describe the strengths and weaknesses of the correlation coefficient as a measure of linear association.
   e) Interpolate using trends observed in scatterplots or fitted regression lines, and judge when extrapolating observed trends may be appropriate.
   f) Given a set of data or scatter plot, choose the line of best fit and use the line to make predictions.
   g) Recognize how linear transformations of one-variable data affect mean, median, mode and range.

2. Describe the influence of outliers on correlation and on models for trend.
   a) Identify unusual observations in scatterplots, and conjecture about the effect of such outliers on the strength of the association between the unknowns defining the scatterplot.
   b) *Use technology to investigate and describe the influence outliers may have on a correlation coefficient, on the slope and intercept of a least-squares regression line, and on a median-fit line.

INTEGRATED MATHEMATICS 2

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Integrated Mathematics 2, Core Content A: Systems of Linear Equations and Matrices 25%

Students represent relationships that can be modeled by a system of linear equations and solve the system using a variety of methods and representations. They represent and interpret data and systems of equations through matrix representations, using addition and multiplication of matrices as appropriate. They use matrix equations and inverses, where they exist, to find solutions to systems of equations using technology. Students extend their understanding of the real number system through solving problems in algebraic situations.

Integrated Mathematics 2, Core Content B: Geometric Proof, Similarity, and Transformations 30%

Students apply and analyze transformations of figures in the coordinate plane using properties of similarity and congruency and apply the concepts of inductive and deductive reasoning to form or verify conjectures. They develop general methods of proof and apply these methods to solve problems involving congruence, similarity, rigid transformations, and origin-centered dilations of figures in the plane.

Integrated Mathematics 2, Core Content C: Direct and Indirect Measurement 20%

Students justify and apply the measurement formulas associated with one-, two-, and three-dimensional geometric objects. They extend their understanding of the real number system through solving problems in geometric situations.

Integrated Mathematics 2, Core Content D: Conditional Probability and Independence 10%

Students determine the sample space for multi-stage experiments. They distinguish between independent and dependent events and compute their probabilities. Students design and use simulations to solve contextual probability problems.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Integrated Mathematics 2, Core Content A: Systems of Linear Equations and Matrices 25%

Students represent relationships that can be modeled by a system of linear equations and solve the system using a variety of methods and representations. They represent and interpret data and systems of equations through matrix representations, using addition and multiplication of matrices as appropriate. They use matrix equations and inverses, where they exist, to find solutions to systems of equations using technology. Students extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Represent relationships that can be modeled by a system of linear equations and solve the system using a variety of methods and representations.
   a) Construct a system of linear equations modeling a contextual situation.
   b) Analyze, solve, and explain the reasoning used to solve a system of linear equations and describe the nature of the solutions (no solution, one solution, infinitely many solutions).

2. Represent and interpret cross-categorized data in matrices, develop properties of matrix addition, and use matrix addition and its properties to solve problems.
   a) Represent numerical or relational data categorized by two or more unknowns in a matrix and label the rows and columns. Interpret the meaning of a particular entry in a matrix in terms of the labels of its row and column.
   b) Use matrix row and column sums to analyze data.
   c) Develop the properties of matrix addition, and add and subtract matrices to solve problems.

3. Multiply matrices, verify the properties of matrix multiplication, and use the matrix form for a system of linear equations to structure and solve systems consisting of two or three linear equations in two or three unknowns, respectively, with technology.
   a) Verify the properties of matrix multiplication and multiply matrices to solve problems.
   b) Construct a system of linear equations modeling a contextual situation, and represent the system as a matrix equation ($Ax = b$), that is:

      i) $$\begin{aligned} ax + by &= c \\ dx + ey &= f \end{aligned} \quad \Leftrightarrow \quad \begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$$

   c) Solve a system consisting of two or more linear equations in two or more unknowns, respectively, by solving the related matrix equation $Ax = b$, using technology to find $x = A^{-1}b$.

Integrated Mathematics 2, Core Content B: Geometric Proof, Similarity, and Transformations 30%

Students apply and analyze transformations of figures in the coordinate plane using properties of similarity and congruency and apply the concepts of inductive and deductive reasoning to form or verify conjectures. They develop general methods of proof and apply these methods to solve problems involving congruence, similarity, rigid transformations, and origin-centered dilations of figures in the plane.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Apply mathematical methods of proof to develop justifications for basic theorems of Euclidean geometry.
   a) Form conjectures based on exploring geometric situations with and without technology.
   b) Prove, directly or indirectly, that a valid mathematical statement is true. Develop a counterexample to refute an invalid statement.
   c) Formulate and investigate the validity of the converse, inverse, or contrapositive of a conditional statement.
   d) Organize and present direct and indirect proofs using two-column, paragraph, and flow-chart formats.

2. Identify congruent figures and justify these congruencies by establishing sufficient conditions and by finding a congruence-preserving transformation between the figures. Solve problems involving congruence in a variety of contexts.
   a) Analyze figures in terms of their symmetries using the concepts of reflection, rotation, and translation and combinations of these.
   b) Compare and contrast equality, congruence, and similarity.
   c) Identify and differentiate among sufficient conditions for congruence of triangles (SSS, SAS, ASA, AAS, and HL).
   d) Use coordinate geometry and rigid transformations (reflections, translations, and rotations) to establish congruence of figures.

3. Identify and apply transformations of figures in the coordinate plane and discuss the results of these transformations.
   a) Represent translations, line reflections, rotations, and origin-centered dilations of objects in the coordinate plane by using sketches, coordinates, and function notation, and explain the effects of these transformations.
   b) Recognize and identify corresponding parts of congruent and similar figures after transformation.

4. Identify similar figures and justify similarity by establishing sufficient conditions and by finding a similarity-preserving rigid transformation or origin-centered dilation between the figures. Solve problems involving similarity in a variety of contexts.
   a) Identify conditions (SAS, SSS, and AA) for establishing similarity of triangles, and apply them, noting that congruence is a special case of similarity.
   b) Use similarity to calculate the measures of corresponding parts of similar figures, and apply similarity in a variety of problem-solving contexts within mathematics and other disciplines.
   c) Create a representation of a figure similar to a specified figure given their similarity ratio and use origin-centered dilations to describe and investigate similarities.
   d) Use similar triangles to demonstrate that the rate of change associated with any pair of points on a line is the same.

Integrated Mathematics 2, Core Content C: Direct and Indirect Measurement **20%**

Students justify and apply the measurement formulas associated with one-, two-, and three-dimensional geometric objects. They extend their understanding of the real number system through solving problems in geometric situations.

Learning Goals and Performance (Locally Assessed) Indicators:

1. Develop two- and three-dimensional measurement formulas for perimeter/circumference, area, and volume and apply these formulas and other geometric properties to solve problems involving simple and composite geometric objects.
   a) Derive and use area formulas for quadrilaterals and regular polygons.
   b) Link the surface area of prisms and cylinders to the sum of the areas of their bases and lateral surfaces using planar nets to illustrate and sum the relevant measures.
   c) Identify and find the measures of angles formed by segments in three-dimensional figures, extending right triangle and isosceles/equilateral triangle relationships to study the planar faces of three-dimensional objects.
   d) Apply and relate formulas and solve problems involving area, perimeter, volume, and surface area of pyramids, cones, spheres, and composite figures.
   e) Determine arc lengths of circles and areas of sectors of circles using proportions.
   f) Develop the triangle angle-sum and angle-measure theorems for polygons, and the triangle- and angle-inequality theorems.
   g) Justify and apply statements about angles formed by chords, tangents, and secants in circles and the measures of their intercepted arcs.

Integrated Mathematics 2, Core Content D: Conditional Probability and Independence 10%

Students determine the sample space for multi-stage experiments. They distinguish between independent and dependent events and compute their probabilities. Students design and use simulations to solve contextual probability problems.

Learning Goals and Performance (*Locally Assessed) Indicators:

3. Determine the sample space for multi-stage experiments (e.g., tossing three coins) using a variety of representations and employ systematic counting approaches to determine the number of possible outcomes. Distinguish between independent and dependent compound events, and compute their probabilities using a variety of representations and applying the multiplication rule for probability.
   a) Use set notation to convey information and use Venn diagrams to illustrate given information about sets.
   b) Determine the number of possible outcomes in the sample space for multi-stage experiments using a variety of representations (e.g., tree diagrams, lists) and systematic counting approaches including permutations, combinations, and the multiplication rule for counting (Fundamental Property of Counting).
   c) Distinguish between independent and dependent compound events, and explain the idea of conditional probability using two-way tables and appropriate notation and terminology (e.g. union)
   d) Use tables, trees, and geometric representations (area models) to describe and apply the multiplication rule for probability to compute probabilities for independent and for dependent compound events.

4. Develop, use, and interpret simulations to estimate probabilities for events where theoretical values are difficult or impossible to compute. Recognize that simulation results are likely to differ from one run of the simulation to the next, and that results of the simulation tend to converge as the number of trials increases.
   a) *Describe and conduct a simulation by identifying the components and assumptions in a problem, selecting a device to generate chance outcomes, defining a trial, specifying the number of trials and carrying out the simulation.
   b) Summarize data from a simulation using appropriate graphical and numerical summaries, develop an estimate for the probability of an event where theoretical values are difficult or impossible to compute, and discuss the effect of the number of trials on the estimated probability of the event.

INTEGRATED MATHEMATICS 3

A percentage of instructional time for each core content area is suggested. The sum of the percentages totals less than 100% so that school districts can modify the emphasis and/or add additional content in response to local needs.

Integrated Mathematics 3, Core Content A: Polynomial Expressions, Equations and Functions 30%

Students extend understanding of functions from linear settings to include polynomial functions, systems, operations on these functions, and the solution of polynomial equations using complex numbers. They use polynomials (with a focus on quadratics) to model situations with graphical and symbolic representations. They flexibly move among polynomial representations (e.g. factored form, standard form) to represent and discuss the qualitative behavior of the associated functions.

Integrated Mathematics 3, Core Content B: Exponential, Logarithmic, and Other Functions 25%

Students develop exponential, logarithmic, and other non-linear functions (rational, radical, absolute value, and piecewise-defined) to represent, investigate, and solve problems in context. They extend their understanding of the real number system through solving problems in algebraic situations.

Integrated Mathematics 3, Core Content C: Sequences and Recursion 10%

Students analyze and represent sequences and series and investigate how recursive relationships and their associated sequences can model the long-term behavior of situations involving sequential change. Students extend their understanding of the real number system through solving problems in algebraic situations.

Integrated Mathematics 3, Core Content D: Indirect Measurement 10%

Students prove and apply the Pythagorean Theorem, right-triangle trigonometric ratios, and proportionality relationships in structuring and solving indirect measurement problems. They extend their understanding of the real number system through solving problems in geometric situations.

Integrated Mathematics 3, Core Content E: Binomial Theorem and Probability 10%

Students use the binomial expansion theorem and recognize the connection to Pascal's triangle, probability, and combinatorics. Students recognize a binomial probability setting, develop and graph the binomial probability distribution, and compute the probability distribution for a binomial count. They extend their understanding of the real number system through problem solving situations.

NOTE: The order of the Core Content, Learning Goals, and Performance Indicators on the pages that follow does not prescribe a sequence of instruction. Sequencing should be determined by individual school districts based on their local curricula and instructional programs.

Integrated Mathematics 3, Core Content A: Polynomial Expressions, Equations and Functions 30%

Students extend understanding of functions from linear settings to include polynomial functions, systems, operations on these functions, and the solution of polynomial equations using complex numbers. They use polynomials (with a focus on quadratics) to model situations with graphical and symbolic representations. They flexibly move among polynomial representations (e.g. factored form, standard form) to represent and discuss the qualitative behavior of the associated functions.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Operate with monomials, binomials, and polynomials. Apply the composition of functions to model and solve problems.
   a) Add, subtract, multiply, and divide polynomial expressions.
   b) Analyze and graph polynomial functions by identifying the intercepts, zeros, domain and range, turning points, and end behavior.
   c) Use factoring (including differences of squares, sum and difference of cubes and other factorable polynomials) to transform expressions and solve problems.
   d) Apply the Factor Theorem to solve polynomial equations and the Remainder Theorem to evaluate polynomial functions.
   e) *Apply the Fundamental Theorem of Algebra to determine the number of real and complex roots of a polynomial function.
   f) Apply the composition of functions to model and solve problems, explaining the results.
2. Represent, compare, move flexibly between and among representations (including graphic, symbolic, and tabular) to interpret and solve problems involving quadratic functions. Develop and apply the quadratic formula.
   a) Write, interpret, apply and move flexibly between and among equivalent forms of quadratic expressions, equations and functions, including the general (standard) and vertex forms, recognizing that equivalent forms for a quadratic relationship reveal different information about a given situation.
   b) Determine reasonable domain and range values for quadratic functions within a context, and test the reasonableness of solutions to quadratic equations (zeros of quadratic functions).
   c) *Identify points of intersection of the graph of a quadratic equation of the form $y = ax^2$ and the graph of a line of the form $y = k$, and relate the points of intersection to the solutions of the quadratic equation $ax^2 = k$.
   d) Graph a quadratic function, and recognize the relationships between the coefficients of a quadratic function and characteristics of its graph (e.g., shape, position, intercepts, zeros, maximum, minimum, symmetry, vertex).
   e) Solve quadratic equations and inequalities by factoring, completing the square, and with technology. Interpret and justify solutions in terms of the original problem context and represent solutions graphically.
   f) Apply the quadratic formula to solve quadratic equations and interpret the nature of the roots.
3. Construct, solve, and interpret solutions of systems of equations and inequalities (including all combinations of linear and quadratic equations) in two unknowns.
   a) Construct a system of equalities or inequalities in two unknowns to represent a contextual setting.
   b) *Analyze and explain the reasoning used to solve systems of linear equations and inequalities in two unknowns.
   c) Solve a system of equalities or inequalities in two unknowns using a variety of methods, and interpret the meaning of the solution.
4. Represent and apply properties of complex numbers.
   a) Define, plot, and compute with complex numbers.
   b) *Describe how the associative, commutative, and distributive properties of operations on real numbers extend to operations on complex numbers.
   c) Solve quadratic equations with real coefficients over the set of complex numbers.

Integrated Mathematics 3, Core Content B: Exponential, Logarithmic, and Other Functions 25%

Students develop exponential, logarithmic, and other non-linear functions (rational, radical, absolute value, and piecewise-defined) to represent, investigate, and solve problems in context. They extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Represent geometric or exponential growth with exponential functions and equations, and apply such functions and equations to solve problems in contextual situations.
   a) Simplify expressions with real (both rational and irrational) exponents and relate the expressions with rational exponents to equivalent radical expressions. Describe the effects of changes in the coefficient, base, and exponent on the growth described by an exponential function.
   b) Analyze a problem situation modeled by an exponential function (e.g. exponential growth and decay, compound interest), formulate an equation or inequality, and solve the problem.
   c) Graph and analyze the behavior of exponential functions.
   d) *Identify any points of intersection of the graph exponential equation of the form $y = ab^x$ and the graph of a line of the form $y = k$ with and without technology and relate the points of intersection to the solutions of the exponential equation $y = ab^x$.

2. Define logarithmic functions and use them to solve problems in contextual situations.
   a) Define a logarithm as a solution to an exponential equation, and recognize the inverse relationship between logarithmic functions and exponential functions, showing this relationship graphically.
   b) Solve problems by applying properties of logarithms (for example, $\log_b xy = \log_b x + \log_b y$) to construct equivalent forms of a logarithmic expression.
   c) Apply the inverse relationship between exponential and logarithmic functions to solve contextual problems.

3. Interpret and represent rational and radical functions and solve rational and radical equations.
   a) Model and solve problems using direct, inverse, joint, and combined variation.
   b) Add, subtract, multiply, divide, and simplify rational expressions with linear and quadratic denominators. Evaluate rational functions.
   c) Model problem situations by constructing equations and inequalities based on rational and radical functions, solve using a variety of methods, and interpret solutions in terms of the problem situation.
   d) Describe the graphs of rational and radical functions, including limitations on the domains and ranges, and asymptotic behavior.
   e) Use properties of radicals to compute and simplify radicals, radical expressions, to solve radical equations and to identify extraneous roots when they occur.

4. Interpret and model step and other piecewise-defined (i.e. linear and quadratic) functions, including absolute value functions.
   a) Analyze a problem situation to determine or interpret reasonable domain and range values for piecewise-defined functions representing the situation.
   b) Interpret, construct and apply step functions (e.g., greatest integer/floor) and other piecewise-defined functions, including absolute value functions, to model and solve problems.
   c) *Move flexibly between and among verbal, graphical, tabular, and symbolic representations of step functions and other piecewise-defined functions, including absolute value functions.

Integrated Mathematics 3, Core Content C: Sequences and Recursion **10%**

Students analyze and represent sequences and series and investigate how recursive relationships and their associated sequences can model the long-term behavior of situations involving sequential change. Students extend their understanding of the real number system through solving problems in algebraic situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Categorize sequences as arithmetic, geometric, or neither and develop formulas for the general terms and sums related to arithmetic and geometric sequences.
   a) *Investigate the rate of change found in sequences, and use it to characterize sequences as arithmetic, geometric, or neither.
   b) Develop the general term for arithmetic and geometric sequences and develop methods for calculating sums of terms for finite arithmetic and geometric sequences and the sum of a convergent infinite geometric series.

2. Develop and apply recursive relationships for modeling and investigating patterns in the long-term behavior of their associated sequences.
   a) Develop and apply recursive relationships for arithmetic and for geometric growth situations.
   b) Generate or construct sequences from given recursive relationships modeling patterns found in mathematics and in other disciplines.
   c) *Analyze the long-term behavior of a recursive relationship, with and without technology.

Integrated Mathematics 3, Core Content D: Indirect Measurement **10%**

Students prove and apply the Pythagorean Theorem, right-triangle trigonometric ratios, and proportionality relationships in structuring and solving indirect measurement problems. They extend their understanding of the real number system through solving problems in geometric situations.

Learning Goals and Performance (*Locally Assessed) Indicators:

1. Prove and apply the Pythagorean Theorem and its converse. Develop and apply the distance formula, properties of special right triangles, properties of proportions, and the basic trigonometric ratios.
   a) Apply the distance formula to determine the distance between points in the coordinate plane.
   b) *Apply properties of 30°-60°-90° and 45°-45°-90° triangles and the Pythagorean Theorem to determine triangle side lengths.
   c) Prove and apply the Pythagorean Theorem and its converse.
   d) Use properties of proportionality and similarity in solving contextual problems in two- and three-dimensional settings.
   e) Apply the sine, cosine, and tangent trigonometric ratios to determine lengths and angle measures in right triangles.

Integrated Mathematics 3, Core Content E: Binomial Theorem and Probability 10%

Students use the binomial expansion theorem and recognize the connection to Pascal's triangle, probability, and combinatorics. Students recognize a binomial probability setting, develop and graph the binomial probability distribution, and compute the probability distribution for a binomial count. They extend their understanding of the real number system through problem solving situations.

Learning Goals and Performance (*Locally Assessed) Expectations:

1. Develop facility with the binomial expansion theorem and its connections to Pascal's triangle, probability, and combinatorics. Recognize a binomial probability setting and compute the probability distribution for a binomial count.
   a) *Use the binomial expansion theorem, and relate the expansion of $(a + b)^n$ with the possible outcomes of a binomial experiment and the n^{th} row of Pascal's triangle.
   b) *Recognize a binomial probability setting, and develop and graph the probability distribution for a binomial count.

2. Identify settings in which the normal distribution may be useful, and describe characteristics of the normal distribution. Use graphical displays and the empirical rule to solve problems.
   a) *Identify settings in which the normal distribution may be useful, and describe characteristics of a normal distribution including mean, standard deviation, and z-scores.
   b) *Use graphical displays and the empirical rule to evaluate the appropriateness of the normal model for a given set of data, and use the empirical rule to estimate the probability that an event will occur in a specific interval that can be described in terms of whole numbers of standard deviations about the mean.

NOTES:

Core content, learning goals and performance indicators for two additional courses (Precalulus and Integrated 4) are under development and will be added following public comment and revision.

A glossary of key terms related to this document is under development. The glossary previously prepared for the 2004 GLE document is online at the DESE website. A revised version, adding key terms used in this document will be posted soon.